Math 172 , Spring 2013 **Section:** ______ **Name:** Solns

Midterm 2 : Tuesday, March 5, 2013.

Problem	1	2	3	4	5	6	7	8	Total
Value	10	15	15	15	10	10	15	10	100
Points									

Instructions : All work must be shown to receive full credit.

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1. [10pts] Evaluate the following trigonometric integral:

$$\int_0^{\pi/2} \cos^3 x \, \sin^2 x \, dx$$

$$\int_0^{\pi/2} \sin^2 x \underbrace{(1-\sin^2 x)\cos x}_{\cos^3 x} \, dx \qquad u = \sin x$$

$$\int_0^1 u^2(1-u^2)\,du = \frac{2}{15}$$

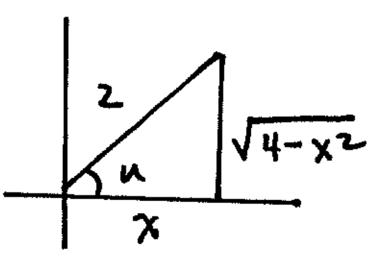
2. [15pts] Use a trigonometric substitution to evaluate (as a function of x):

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

$$x = 2\cos u$$
$$dx = -2\sin u \, du$$

$$= \int \frac{(-2\sin u)\,du}{4\cos^2 u \sqrt{4\sin^2 u}}$$

$$= -\frac{1}{4}\int \sec^2 u \, du = -\frac{1}{4}\tan + C$$



$$= -\frac{1}{4}\frac{\sqrt{4-x^2}}{x} + C$$

3. [15pts] Use partial fraction expansions to evaluate the following integral:

$$\int \frac{2x+4}{x(x^2+4)}\,dx$$

$$\frac{2x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$= \frac{A(x^2+4) + Bx^2 + Cx}{x(x^2+4)}$$

Comparing sides: $A = 1,\ B = -1,\ C = 2$

$$I = \int \frac{1}{x} - \frac{x}{x^2+4} + \frac{2}{x^2+4}\,dx$$

$$I = \ln|x| - \frac{1}{2}\ln|x^2+4| + \arctan\left(\frac{x}{2}\right) + C$$

4. [15pts] The following involves improper integrals.

a) If the following integral is convergent, evaluate it. If not, prove it is divergent.

$$I = \int_0^\infty \frac{x}{x^4+1}\,dx$$

$$I_R = \int_0^R \frac{x}{(x^4+1)}\,dx = \int_0^{R^2} \frac{du}{2(u^2+1)} \qquad u = x^2$$

$$I = \frac{1}{2}\lim_{R\to\infty}\int_0^R \frac{du}{(u^2+1)} = \frac{1}{2}\lim_{R\to\infty} \arctan R = \frac{\pi}{4}$$

b) Use a Comparison Theorem to prove if the following integral is convergent or divergent.

$$\int_1^\infty \frac{1}{\sqrt{x^4+2}}\,dx$$

$$0 < \frac{1}{\sqrt{x^4+2}} < \frac{1}{x^2}$$

Thus

$$0 < \int_1^\infty \frac{dx}{\sqrt{x^4+2}} < \int_1^\infty \frac{dx}{x^2}$$

Converges by Comp. Thm

convergent p-int.

5. [10pts] For some value of C the function $p(x)$ is a probability density function on $(-2, 2)$

$$p(x) = \frac{C}{\sqrt{4 - x^2}}$$

a) Find C.

$$\int_{-2}^{2} \frac{c}{\sqrt{4-x^2}}\,dx = \tfrac{1}{2}c \arcsin\left(\tfrac{x}{2}\right)\Big|_{-2}^{2} = c\pi = 1$$

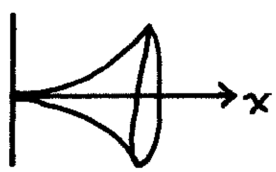
$$\boxed{c = \frac{1}{\pi}}$$

b) Compute the probability $P(0 \le X \le 1)$.

$$P = \int_0^1 \frac{1}{\pi\sqrt{4-x^2}}\,dx = \arcsin\left(\tfrac{x}{2}\right)\Big|_0^1 = \tfrac{1}{6}$$

6. [10pts] Find the surface area formed by revolving the following curve about the x-axis.

$$y = x^3 \quad , \quad 0 < x < 1$$



$$S = 2\pi \int_0^1 f(x)\sqrt{1 + f'(x)^2}\,dx$$

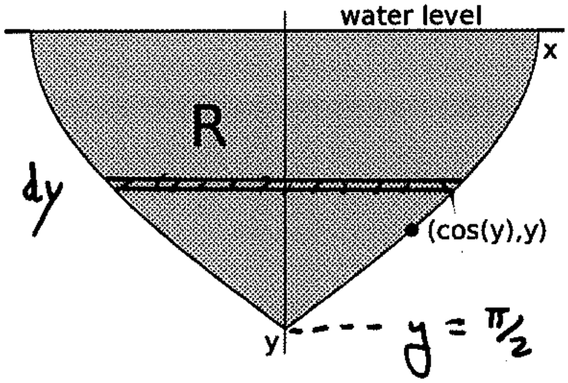
$$S = 2\pi \int_0^1 x^3 \sqrt{1 + 9x^4}\,dx \qquad u = 1 + 9x^4, \quad du = 36\,x^3 dx$$

$$S = \frac{\pi}{18} \int_0^{10} u^{1/2}\,du$$

$$S = \frac{\pi}{27}\left(10^{3/2} - 1\right)$$

7. [15pts] A plate R bounded by the curves $y = 0$, $x = \cos(y)$ and $x = -\cos(y)$ and $y = 0$ is submerged vertically in water (see shaded region in figure below).

a) Set up a definite integral whose value is the fluid force F acting on R. You may leave your answer as a multiple of the product ρg where $\rho = 1000kg/m^3$ is density of water and $g = 9.8m/sec^2$ is the gravitational constant.



$f(y) = 2\cos y$

$$F = \rho g \int_0^{\pi/2} 2y \cos y \, dy$$

b) Compute the force F in part a). Int. By. Parts $u = y, \; dv = \cos y \, d$

$$F = 2\rho g \left(\cos y + y \sin y \right) \Big|_0^{\pi/2}$$

$$F = \pi - 2$$

8. [10pts] Evaluate the indefinite integral below. Hint: first use $u = \cos x$

$$I = \int \frac{dx}{\sin x \ \cos^2 x}$$

$du = -\sin x \, dx$

$$I = \int \frac{-du}{\sin^2 x \ \cos^2 x}$$

$$I = \int \frac{-du}{u^2(1-u^2)}$$

$$I = \int \left\{ \frac{A}{u^2} + \frac{B}{1-u} + \frac{C}{1+u} \right\} du$$

lot's of algebra

$$I = \int -\frac{1}{u^2} + \frac{1}{2(u-1)} - \frac{1}{2(u+1)} \ du$$

$$I = \frac{1}{u} + \frac{1}{2}\ln(u-1) - \frac{1}{2}\ln(u+1)$$

$$I = \sec x + \frac{1}{2}\ln\left(\frac{\cos x - 1}{\cos x + 1}\right) + C$$