Math 172 , Spring 2013 **Section:** ________ **Name:** Solns

Midterm 3 : Tuesday, April 9, 2013.

Problem	1	2	3	4	5	6	7	8	Total
Value	10	10	15	15	15	15	10	10	100
Points									

Instructions : All work must be shown to receive full credit.

1. [10pts] Compute the sum of the following convergent Geometric Series:

$$S = \sum_{n=2}^{\infty} \frac{4}{5^n} = \frac{4}{5^2} + \frac{4}{5^3} + \frac{4}{5^4} + \cdots$$

$$= \frac{4}{25}\left(1 + \frac{1}{5} + \frac{1}{5^2} + \cdots\right)$$

$$= \frac{4}{25} \frac{1}{1 - 1/5}$$

$$= \frac{1}{5}$$

2. [10pts] Use the Integral Test to determine if the following series converges or diverges. You may assume all hypotheses are satisfied.

$$S = \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3} \qquad f(x) = \frac{1}{x(\ln x)^3}$$

$$I = \int_2^{\infty} \frac{dx}{x(\ln x)^3} = -\frac{1}{2(\ln x)^2}\Big|_2^{\infty} \qquad u = \ln x$$

$$= \lim_{R\to\infty} -\frac{1}{2(\ln x)^2}\Big|_2^{R} = \frac{1}{2}(\ln 2)^{-2}$$

Since the integral I converges, so does the sum S.

3. [15pts] State the Ratio test with all its possible conclusions and then use it to determine if the following series converges.

$$S = \sum_{n=1}^{\infty} \frac{e^n}{n!} \qquad a_n = \frac{e^n}{n!}$$

Suppose $\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$ exists. Then

(i) $\rho < 1 \Rightarrow S$ converges absolutely

(ii) $\rho > 1 \Rightarrow S$ diverges

(iii) $\rho = 1$ test inconclusive

Here

$$\frac{a_{n+1}}{a_n} = \frac{e^{\cancel{n}+1}}{(n+1)!} \frac{\cancel{n!}}{e^{\cancel{n}}} = \frac{e}{n+1} \to 0$$

S converges absolutely.

4. [15pts] Use any comparison test to determine if the following series converges or diverges.

$$S = \sum_{n=2}^{\infty} \frac{n}{\sqrt{n^4 + 2n^2}} \qquad a_n \sim \frac{n}{\sqrt{n^4}} = \frac{1}{n}$$

Limit comparison test with $b_n = \frac{1}{n}$.

$$\frac{a_n}{b_n} = \frac{n^2}{\sqrt{n^4 + 2n^2}} = \frac{1}{\sqrt{1 + \frac{2}{n^2}}} \to L = 1$$

Since $L > 0$ and Σb_n diverges (p-series) then $S = \Sigma a_n$ diverges by LCT.

5. [15pts] Answer all three parts:

a) Does the series below satisfy all the hypotheses of the Alternating Series Test? Why or why not?

$$S = \sum_{n=1}^{\infty} \frac{(-1)^n \sin(n)}{n^2} \quad = (-1)^n a_n$$

<u>No</u> $\quad a_n$ changes sign and is not $\downarrow$.

b) Determine whether the series in a) is absolutely convergent.

$$b_n = \frac{\sin(n)}{n^2}$$

Converges Absolute.

$$0 \le |b_n| \le \frac{1}{n^2}$$

and $p=2$ converg. series

$$0 \le \sum |b_n| \le \sum \frac{1}{n^2}$$

c) The series below satisfies all the hypotheses of the Alternating Series Test:

$$S = \sum_{n=0}^{\infty} (-1)^n a_n = \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+4}$$

Find a numerical bound on the error in approximating S by the first four terms, i.e.,

$$a_n = \frac{1}{n^2+4} \qquad a_4 = \frac{1}{20}$$

$$|S - (a_0 - a_1 + a_2 - a_3)| < \underline{\quad \frac{1}{20} \quad}$$

6. [15pts] Let

$$f(x) = x^2 \ln(1 + 4x^2)$$

. Use the attached tables to:

a) Find the first three nonzero terms of the Taylor series of $f(x)$ about $x = 0$

$$f(x) = x^2 \left((4x^2) - \frac{1}{2}(4x^2)^2 + \frac{1}{3}(4x^2)^3 - \cdots \right)$$

$$f(x) = 4x^4 - 8x^6 + \frac{64}{3}x^8 + \cdots$$

Derived using $u = 4x^2$ in

(1) $$\ln(1+u) = u - \frac{1}{2}u^2 + \frac{1}{3}u^3 - \cdots$$

for $|u| < 1$ and $u = 1$.

$f(x) =$ ____________________________________

b) Determine the interval of convergence:

$$u = 4x^2 = 1 \quad \Rightarrow \quad x = \pm \frac{1}{2}$$

Interval of Convergence = $[-\frac{1}{2}, \frac{1}{2}]$

7. [10pts] For the following statements, circle if they are True or False.

a) If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges.

$a_n = \frac{1}{n}$

TRUE FALSE

b) If $\sum_{n=1}^{\infty} a_n$ converges absolutely then $\sum_{n=1}^{\infty} a_n$ converges.

TRUE FALSE

c) The series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$ diverges.

$a_n \to 1 \quad \text{as } n \to \infty$

TRUE FALSE

d) If $a_n < b_n$ for all n and $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

$b_n = \frac{1}{n^2}$

$a_n = -\frac{1}{n}$

TRUE FALSE

e) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} b_n$ converges.

$a_n = \frac{1}{n^2}$, $\sum \frac{1}{n^2}$ conv.

$b_n = \frac{1}{n}$, $\sum \frac{1}{n}$ div

TRUE FALSE

8). [10pts] (Power Series)

i) Use any convergence theorem to determine the radius of convergence R of

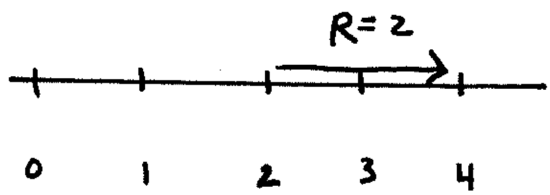
$$S = \sum_{n=1}^{\infty} \left(\frac{\sqrt{n}}{2\,(1+\sqrt{n})} \right)^{n} (x-2)^{n} \quad = a_n$$

ROOT TEST

$$\lim_{n \to \infty} \sqrt[n]{|a_n|} = \lim_{n \to \infty} \frac{\sqrt{n}}{2(1+\sqrt{n})} \cdot |x-2|$$

$$= \frac{1}{2} |x-2|$$

Converge absolutely if $|x-2| < \underline{\underline{2}}$



$R =$ 2

ii) Does the series converge when $x = 3$? Why or why not?

$x = 3$ is in the interval of Convergence above.

Series S converges for $x = 3$.