Math 333 (2005) Assignment 2

(Due: September 22, 2005 in class)

1. (10) For each of the linear transformations T defined below, determine if T is 1-1 and if it is find $[T^{-1}]$.

a) $T(\mathbf{x}) = (x_1 + 2x_2, -x_1 + x_2)$, $\mathbf{x} = (x_1, x_2)$

b) $T(\mathbf{x}) = (x_1 - 2x_2 + 2x_3, 2x_1 + x_2 + x_3, x_1 + x_2)$, $\mathbf{x} = (x_1, x_2, x_3)$

2. (10) For each of the linear transformations T defined below, describe the range $R(T)$ and state whether T is 1-1.

a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ is the projection onto $u = (1, 2, 3)$.

b) $T : \mathbb{R}^3 \to \mathbb{R}$ where $T(\mathbf{x}) = x_1 + x_2 - x_3$.

3. (15) For each of the linear transformations T defined below, use properties a) and b) in Theorem 4.3.2 of the textbook to determine if T is a linear transformation. If T is not linear, state which property(ies) is(are) violated.

a) $T(\mathbf{x}) = (x_1 + 1, x_2)$, $\mathbf{x} = (x_1, x_2)$

b) $T(\mathbf{x}) = (x_1, x_2 + x_3)$, $\mathbf{x} = (x_1, x_2, x_3)$

c) $T(\mathbf{x}) = (1, 1)$, $\mathbf{x} = (x_1, x_2)$

4. (15) State whether each of the following statements are true or false and give a simple explanation or counterexample.

a) The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which rotates vectors and then projects them onto $u = (1, 2)$ is invertible.

b) If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a 1-1 linear transformation then $m = n$.

c) If $T : \mathbb{R}^n \to \mathbb{R}^m$ and $T(\mathbf{0}) = \mathbf{0}$ then T is linear.

5. (10) For each of the linear transformations T defined below, use Theorem 4.3.3 of the textbook to determine $[T]$.

a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation which rotates vectors clockwise by $30°$ and then projects onto the x_2 axis.

b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation which projects vectors onto $u = (1, 2)$ and then dilates by 2.

6. (15) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator.

a) If $T(2\mathbf{e}_1) = (2, 2, 4)$ and $T(\mathbf{e}_1 - \mathbf{e}_2) = (1, -1, 0)$ what is $T(4\mathbf{e}_1 - 5\mathbf{e}_2)$?

b) If additionally one knows $T(\mathbf{e}_3) = (1, 3, 4)$ then what is $[T]$?

c) Is T 1-1?