Math 333 (2005) Assignment 4

(Due: October 13, 2005 in class)

Maximum 45 points

1. (15) In the following problems compute the coordinate $(w)_S$ of $w \in V$ relative the indicated bases. If, for example,

$$S = \{v_1, v_2, v_3\}$$

where the vectors v_k are defined then the coordinate $(w)_S$ is that vector $c = (c_1, c_2, c_3) \in \mathbb{R}^3$ such that $w = c_1 v_1 + c_2 v_2 + c_3 v_3$.

a) $V = \mathbb{R}^2$, $v_1 = (1, 2)$, $v_2 = (-1, 1)$, $S = \{v_1, v_2\}$, $w = (1, 1)$

b) $V = P_2$, $v_1 = x^2 + 1$, $v_2 = x + 1$, $v_3 = x - 1$,$S = \{v_1, v_2, v_3\}$, $w = x^2 + x + 1$.

2. (10) Determine a basis for the following vector spaces W:

a) $W = \left\{x \in \mathbb{R}^3 : 2x_1 + x_3 = 0\right\}$

b) The solution space $W \subset \mathbb{R}^4$ of

$$\begin{aligned} 2x_1 + x_2 - x_3 + x_4 &= 0 \\ x_1 - x_2 - x_3 + 2x_4 &= 0 \end{aligned}$$

3. (10) Let S be the set of polynomials

$$S = \{x^3, x^2 + x, x^3 + x^2 + x, x^3 - x\}$$

and $W = span(S)$. Determine a basis S' for W which contains only elements of S and prove (i) S' is an independent set and (ii) $W = span(S')$.

4. (10) Let $V = M_{22}$ and define

$$v_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad , \; v_2 = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \quad , \; v_3 = \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix} .$$

Show that $S = \{v_1, v_2, v_3\}$ is a dependent set in V by showing that

$$S' = \{(v_1)_E, (v_2)_E, (v_3)_E\}$$

is a dependent set in $\mathbb{R}^4$ where E is the standard basis for M_{22}.

5. (0 points) If your answer to this question is correct then are you correct?