Math 333 (2005) Assignment 6

(Due: November 15, 2005 in class)
Maximum 50 points

1. (30) For each of the indicated inner product spaces V, subspaces W and vector u, compute the projection $w = proj_W u$ and $w^\perp$ in the orthogonal decomposition

$$u = w + w^\perp$$

a)

$$\begin{aligned} V &= \mathbb{R}^2 \quad , \quad < u, v > \equiv (Au)^T(Av) \quad , \quad A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} \\ u &= (1,1) \quad , \quad W = span\{(3,1)\} \end{aligned}$$

b)

$$\begin{aligned} V &= P_3 \quad , \quad < u, v > \equiv \int_0^1 u(x)v(x)dx \quad , \\ u &= x^3 - x \quad , \quad W = span\{x^3, x^2 + x\} \end{aligned}$$

c)

$$\begin{aligned} V &= M_{22} \quad , \quad < u, v > \equiv Tr(u^T v) \quad , \\ u &= \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \quad , \quad W = span\left\{ \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \right\} \end{aligned}$$

2. (5) Determine an <u>orthonormal</u> basis $S = \{v_1, v_2, v_3\}$ for $V = \mathbb{R}^3$ which contains only vectors in $row(A)$ and $N(A)$ if

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

3. (15) State if the following are True or False. When False, give a counterexample. When true explain why (carefully).

a) If X and Y are subspaces of a vector space V and $V = X + Y$ then $V = X \oplus Y$.

b) If $A \in \mathbb{R}^{n\times n}$ then $\mathbb{R}^n = row(A) \oplus col(A)$.

c) If $v_1 = proj_{.W} u$ and $v_2 = proj_{.W^\perp} u$ then $u = v_1 + v_2$.