Math 333 (2005) Assignment 7

(Due: November 22, 2005 in class)

Maximum 60 points

1. (10) Both of the following matrices have characteristic polynomial

$$P(\lambda) = det(A - \lambda I) = -\lambda^3$$

For each matrix define the eigenspace $E_0(A)$ and then state both the algebraic and geometric multiplicity of the sole eigenvalue $\lambda = 0$.

a)

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

2. (10) Determine if the following matrices are diagonizable. State your reasons why or why not (appeal to a Theorem in the textbook). If they are diagonizable do not determine the matrix P which diagonalizes A.

a)

$$A = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

3. (10) Diagonalize the matrix

$$A = \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}$$

Specifically, determine the matrix P (and it's inverse P^{-1}) such that

$$A = P\Lambda P^{-1}$$

where Λ is a diagonal matrix having the eigenvalues of A along the diagonal. State what P and Λ are.

4. (10) Orthogonally diagonalize the symmetric matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

Specifically, determine the orthogonal matrix Q (and it's inverse $Q^{-1} = Q^T$) such that

$$A = Q\Lambda Q^T$$

where Λ is a diagonal matrix having the eigenvalues of A along the diagonal. State what Q and Λ are.

5. (10) Let

$$A = \begin{bmatrix} -3 & -2 \\ 4 & 3 \end{bmatrix}$$

Determine A^{100} exactly.

6. (10) If Q is an orthogonal matrix and $y = Qx$ then

$$\| y \|^2 = y^T y = (Qx)^T (Qx) = x^T Q^T Q x = x^T I x = x^T x = \| x \|^2$$

implies that x and y have the same length.

Now let $A = A^T \in \mathbb{R}^{n \times n}$ and define the vector x_k by the recurrence relation

$$x_k = A^k x_0$$

where x_0 is any vector in $\mathbb{R}^n$. Further let $\lambda_i, i = 1, 2, \ldots$ be the eigenvalues of A.

(i) If $|\lambda_i| < 1, \forall i$ prove that $\| x_k \| \to 0$ as $k \to \infty$.
(Hint: $A = Q\Lambda Q^T, y = Q^T x$).

(ii) Suppose now that $\lambda_1 = 1$ and $x_0 \in E_{\lambda_1}(A)$, $\| x_0 \| = 1$. Determine:

$$\lim_{k \to \infty} \| x_k \|$$