Math 333 (2005) Homework 8

(Due: December 6, 2005 in class)

Maximum: 55 points

1. (25) Decide which of the following transformations are linear:

a) $T : \mathbb{R}^2 \to \mathbb{R}^2$

$$T(x) = \| x \| \quad , \quad x = (x_1, x_2)$$

b) $T : P_2 \to P_4$

$$T(u) = x^2 u(x) \quad , \quad u(x) \in P_2$$

c) $T : M_{22} \to P_4$

$$T(u) = u_{11}x^4 + u_{12}x^3 + u_{21}x + u_{22} \quad , \quad u = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

d) $T : P_2 \to P_2$

$$T(u) = u(x-1) + u(x+1) \quad , \quad u(x) \in P_2$$

e) $T : P_2 \to P_4$

$$T(u) = u(x)^2 \quad , \quad u(x) \in P_2$$

2. (10) Find a basis for the kernel $ker(T)$ of the following linear transformations:

a)

$$T : \mathbb{R}^2 \to \mathbb{R}^2 \quad , \quad T(x) = (x_1 - x_2, 5x_2 - 5x_1) \quad , \quad x = (x_1, x_2)$$

b)

$$T : P_2 \to \mathbb{R} \quad , \quad T(u) = \int_{-1}^{1} u(x)dx \quad , \quad u(x) = ax^2 + bx + c \in P_2$$

3. (20) In each of the following a linear transformation $T : X \to Y$, and bases for X and Y are defined:

$$B = \{u_1, u_2 \ldots u_n\} \quad , \quad X = span(B)$$

$$B' = \{v_1, v_2 \ldots v_m\} \quad , \quad Y = span(B')$$

Determine the matrix A for which

$$A(x)_B = (T(x))_{B'} \quad , \quad \forall x \in X$$

i.e., the matrix which generates T in coordinate space.

SEE OTHER SIDE

a) $T : P_2 \to P_2$

$$\begin{aligned} T(u) &= u(2x+1) = a(2x+1)^2 + b(2x+1) + c \\ u(x) &= ax^2 + bx + c \\ B &= B' = \{u_1, u_2, u_3\} = \{1, x, x^2\} \end{aligned}$$

b) $T : \mathbb{R}^2 \to \mathbb{R}^2$

$$\begin{aligned} T(x) &= (x_1 + x_2, x_1 - x_2) \\ x &= (x_1, x_2) \\ B &= \{u_1, u_2\} = \{(1,0), (0,1)\} \\ B' &= \{v_1, v_2\} = \{(1,1), (1,-1)\} \end{aligned}$$