Math 333 (2015) - Homework 3

Due: October 15, 2015. NAME: ______________________

1. (10) State which of the following sets V are vector spaces. If it is not a vector space, state <u>all</u> the axioms (A1)-(A10) that are <u>not</u> satisfied.

a) Matrix addition and scalar multiplication and

$$V = \left\{ u \in M_{22} : u = \begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} \text{ for some } a, b \in \mathbb{R} \right\}$$

b) The set $V = \mathbb{R}^2$ where if $x, y \in V$ and $\alpha \in \mathbb{R}$ addition and scalar multiplication are defined by:

$$\begin{aligned} x \oplus y &= (x_1 y_1, x_2 y_2) \\ \alpha \odot x &= (\alpha x_1, \alpha x_2) \end{aligned}$$

2. (20) Determine if each W below is closed under addition and scalar multiplication. Then state if W is/is not a subspace of the associated vector space V.

a) $W = \left\{ u \in M_{22} : u = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ where } a + b + c + d = 0 \right\} \quad , V = M_{22}$

b) $W = \{f \in C(\mathbb{R}) : f(x) \leq 0\} \quad , V = C(\mathbb{R})$

c) $W = \{f \in C(\mathbb{R}) : f(0) = 0\} \quad , V = C(\mathbb{R})$

d) $W = \{A \in M_{nn} : A^T = -A\} \quad , V = M_{nn}$

3. (15) For each of the following express w as a linear combination of the vectors v_k indicated. The vector space V which w, v_k are elements of are also indicated.

a) $V = \mathbb{R}^3$, $w = (7, 8, 9)$, $v_1 = (2, 1, 4)$, $v_2 = (1, -1, 3)$, $v_3 = (3, 2, 5)$.

b) $V = P_2$, $w = 5x^2 + 13x + 3$, $v_1 = x^2 + 2x + 3$, $v_2 = -x^2 - 3x + 1$.

c) $V = M_{22}$, $w = \begin{bmatrix} -1 & -5 \\ 4 & 1 \end{bmatrix}$, $v_1 = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

4. (10) In the following problems compute the coordinate $(w)_S$ of $w \in V$ relative the indicated bases. If, for example,

$$S = \{v_1, v_2, v_3\}$$

where the vectors v_k are defined then the coordinate $(w)_S$ is that vector $c = (c_1, c_2, c_3) \in \mathbb{R}^3$ such that $w = c_1 v_1 + c_2 v_2 + c_3 v_3$.

a) $V = \mathbb{R}^2$, $v_1 = (1, 2)$, $v_2 = (-1, 1)$, $S = \{v_1, v_2\}$, $w = (1, 1)$

b) $V = P_2$, $v_1 = x^2 + 1$, $v_2 = x + 1$, $v_3 = x - 1$,$S = \{v_1, v_2, v_3\}$, $w = x^2 + x + 1$.

5. (5) Let S be the set of polynomials

$$S = \{x^3, x^2 + x, x^3 + x^2 + x, x^3 - x\}$$

and $W = span(S)$. Determine a basis S' for W which contains only elements of S. What is $dim(W)$?