Math 430 Mathematical Biology – Homework 1

Due: Tuesday, February 7, 2023. NAME: ____________________

(1) [15pts] Recall the simple *growth with harvesting model*

$$x_{n+1} = \lambda x_n - h$$

where x_n is the chicken population after n years, x_0 is the initial population, λ is the growth rate and h is the harvesting rate (chickens per year).

a) Find a formula for the harvesting rate h^* at which the population remains constant, i.e., $x_{n+1} = x_n = x_0$ for all n.

b Find a formula for the number of years $\bar{n}$ it takes for the population to die out when it is over harvested, i.e. $h > h^*$.

c) Let $\lambda = 1.05$, $x_0 = 100$, $h = 7$. Use Matlab to simulate the run using n between 0 and 30 with x and y plotting ranges in [0,30], [0,300], respectively. Use your formula in b) to verify the $\bar{n}$ at which the population dies out. Label your calculated value $\bar{n}$ as a point on the n versus x_n plot. Include a hardcopy of the plot and one of the Matlab code.

(2) [15pts] Find the (real) general solution of each of the following difference equations.

$$\begin{aligned} 2x_{n+2} - 7x_{n+1} + 3x_n &= 0 \\ x_{n+2} - 4x_{n+1} + 4x_n &= 0 \\ x_{n+2} - \sqrt{3}x_{n+1} + x_n &= 0 \end{aligned}$$

One characteristic polynomial has two real distinct roots, another one real repeated root and the other has a complex roots with a simple polar representation $\lambda = re^{i\theta}$.

(3) [10pts] Read the textbook description of the Red Blood Cell (RBC) population model in Section 1.9, Problem 2:

$$R_{n+1} = (1-f)R_n + M_n \quad (1)$$

$$M_{n+1} = \gamma f R_n \quad (2)$$

a) Convert the system above model to a single equation involving only R_{n+1}, R_n, R_{n-1}.

b) Show that the eigenvalues of the difference equation in a) are

$$\lambda_{\pm} = \frac{(1-f) \pm \sqrt{(1-f)^2 + 4\gamma f}}{2}$$

c) Homeostasis is when the number of red blood cells R_n is roughly constant. This can be achieved when $\lambda_+ = 1$. Show that for this to happen, γ must equal 1.

d) With $\lambda_+ = 1$ in c), show $\lambda_- = -f$ and then write down the general solution for R_n. What does R_n approach as $n \to \infty$.