Math 430 Mathematical Biology – Homework 2

Due: Tuesday, February 21, 2023.   NAME: _____

1 [10pts] For the following difference equation:

$$x_{n+1} = f(x_n) = -x_n^2(1 - x_n)$$

  i) Compute the first 3 terms of the orbit

$$\gamma(x_0) = \{x_0, x_1, x_2, \ldots\}$$

    using $x_0 = \frac{1}{2}$. Write out x_0, x_1, x_2 as exact fractions.

  ii) Find all fixed points $\bar{x}$ of the map. Note: some may be negative.

  iii) Determine the stability of each fixed point.

2 [20pts] A frequently encountered model of fish population is given by Ricker's equation (see Greenwell 1984):

$$x_{n+1} = \alpha x_n e^{-\beta x_n}$$

Here x_n is the population in thousands at year n, $\alpha > 0, \beta > 0$.

  a) Find the sole positive fixed point $\bar{x} > 0$. For what α is this fixed point positive?

  b) Show that $\bar{x}$ is stable only if the growth rate α satisfies:

$$|1 - \ln \alpha| < 1$$

    This is equivalent to $\alpha \in (\alpha_1, \alpha_2)$. What are α_k?

  c) Modify the posted code cobweb.m to create 3 cobweb figures of the model showing an attraction to a fixed point, an attraction to a period 2 orbit, and an attraction to a period 4 orbit.

  d) What is the maximum possible value of x_n? I want the formula involving α and β. Hint: What's the maximum value of $f(x)$ for $x > 0$?