Math 430 Mathematical Biology – Homework 3

Due: Tuesday, March 7, 2023. NAME: ____________________

1) [20] For each of the four linear systems

i) find the general solution,

ii) classify the type of equilibria the origin is (saddle, center, etc)

iii) use pplane9.m to create a phase portrait for $-2 < x < 2$, $-2 < y < 2$

$$\frac{d\vec{x}}{dt} = A\vec{x} = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \vec{x} \tag{1}$$

$$\frac{d\vec{x}}{dt} = A\vec{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \vec{x} \tag{2}$$

$$\frac{d\vec{x}}{dt} = A\vec{x} = \begin{bmatrix} -4 & -17 \\ 2 & 2 \end{bmatrix} \vec{x} \tag{3}$$

$$\frac{d\vec{x}}{dt} = A\vec{x} = \begin{bmatrix} 1 & 1 \\ -17 & -1 \end{bmatrix} \vec{x} \tag{4}$$

2) [10] For the following two nonlinear systems

i) Find all equilibria

ii) Use Figure 5.14 of the text ($trA, det(A)$ diagram) to classify each equilibria's type (saddle,...) Be careful with the second system especially when considering $x = 0$.

$$\frac{dx}{dt} = x^2 - y \qquad \frac{dy}{dt} = x - 1$$

$$\frac{dx}{dt} = x(1 - x) \qquad \frac{dy}{dt} = y\left(1 - \frac{y}{x}\right)$$

3) [10] The dimensionless chemostat model is:

$$\frac{dn}{dt} = \alpha_1 \frac{nc}{1 + c} - n \tag{5}$$

$$\frac{dc}{dt} = -\frac{nc}{1 + c} - c + \alpha_2 \tag{6}$$

i) The coexistence equilibria is physical only if (α_1, α_2) satisfy two inequalities (see posted notes or text for these). These in turn define a region in the (α_1, α_2)-plane. Accurately draw (sketch or shade) this region (along with its bounding curves) only for positive (physical) α_k.

ii) Determine the equality (or equalities) which (α_1, α_2) must satisfy for the extinction state (of bacteria) to be stable. As in i), draw/sketch the region in the (α_1, α_2)-plane where the extinction state is stable.