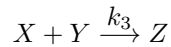
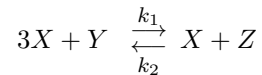


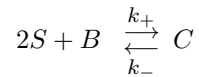
Math 430 Mathematical Biology – Homework 5

Due: Tuesday, April 18, 2023. NAME: _____

1) [5] Use Law of Mass Action to write out the (molar) concentration differential equations for the following reactions.



2) [10] Let S =substrate, B =buffer and C =complex. A slightly different form of buffering is given by the reaction equations:



Using the Law of Mass Action the differential equations governing the molar concentrations is given by:

$$\frac{dS}{dt} = -2k_+BS^2 + 2k_-C \quad , \quad S(0) = S_0 \quad (1)$$

$$\frac{dB}{dt} = -k_+BS^2 + k_-C \quad , \quad B(0) = B_0 \quad (2)$$

$$\frac{dC}{dt} = k_+BS^2 - k_-C \quad , \quad C(0) = C_0 \quad (3)$$

a) Find the function $f(S, B)$ such that the above system can be reduced to

$$S' = 2f(B, S)$$

$$B' = f(B, S)$$

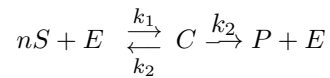
b) Find the function $g(S)$ such that $f = 0$ only if

$$B = g(S) = \frac{\bar{V}}{K_E + S^2}$$

for certain parameters \bar{V} and K_E (write these out in terms of the original model parameters : k_-, k_+, S_0, B_0)

- c) Use Matlab or any other plotting program to make a phase plane portrait like that done in my lecture notes for the regular buffering problem. It should be in the (S,B)-plane, be first quadrant only, have an accurate plot of $f = 0$ and several trajectories. Use the parameter values $k_+ = k_- = S_0 = B_0 = 1$ and S values in $[0, 2]$

3) [10] A variation of Michaelis-Menten kinetics is n -site cooperativity where n substrate molecules S must bind in order for the product P to be made. The reaction equations are:



The subsequent dimensional differential equations for the reaction are

$$\begin{aligned} \frac{dS}{dt} &= f_1 = -nk_1S^nE + nk_{-1}C & , & & S(0) = S_0 \\ \frac{dE}{dt} &= f_2 = -k_1S^nE + (k_{-1} + k_2)C & , & & E(0) = E_0 \\ \frac{dC}{dt} &= -f_2 = k_1S^nE - (k_{-1} + k_2)C & , & & C(0) = 0 \\ \frac{dP}{dt} &= k_2C \end{aligned}$$

- a) Without nondimensionalizing the equations, use conservation of receptors to find the quasi-steady state approximation of the production rate

$$\frac{dP}{dt} = V(S)$$

that only depends on S (and not E, C). This is the ubiquitous "Hille" function.

- b) According to the fast-subsystem analysis (in class and notes), the complex concentration very quickly rises from 0 to a maximum value C_{max} . Find a formula for C_{max}