Math 450 (2017) – Homework 1

Due: Wed. September 13, 2017. NAME: ______________________________

1. [20pts] Find the solution of the following initial value problems:

\[ y' - \frac{1}{2x} y = \sqrt{x} \quad , \quad y(1) = 4 \quad (1) \]
\[ y' + \frac{1}{x} y = \frac{5x}{y^2} \quad , \quad y(1) = 2 \quad (2) \]
\[ y'' - 2y' + 5y = 0 \quad , \quad y(0) = 0 \quad , \quad y'(0) = 1 \quad (3) \]
\[ y'' + 2y' + y = 1 \quad , \quad y(0) = 1 \quad , \quad y'(0) = 2 \quad (4) \]

2. [5pts] Use the method of Variation of Parameters to find a particular solution of

\[ y'' - 2y' + y = x^3 e^x \]

3. [5pts] Let \( f(x, y) = y - \log(x) \). Find that unique curve through \((x, y) = (1, 0)\) that is orthogonal to the level curves of \( f \). Sketch several \( f = c \) level curves and the resulting orthogonal curve (just for \( x > 0 \)).

4. [10pts] Find a Fundamental Matrix \( X(t) \in \mathbb{R}^{2 \times 2} \) of the system

\[ \frac{dx}{dt} = Ax \]

for the following two matrices:

\[ A = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} \quad , \quad A = \begin{bmatrix} 2 & -5 \\ 1 & 0 \end{bmatrix} \]