Math 450 (2023) – Homework 2

Due: Monday, October 2, 2023. NAME: ____________________

1. [6pts] Let $x = \bar{x}(\epsilon)$ be the root of

$$x^5 - \epsilon x - 32 = 0$$

and assume

$$\bar{x}(\epsilon) = x_0 + x_1\epsilon + x_2\epsilon^2 + O(\epsilon^3)$$

Using the Binomial Theorem one can show

$$\bar{x}(\epsilon)^5 = x_0^5 + 5x_0^4 x_1\epsilon + \left(5x_0^4 x_2 + 10x_1^2 x_0^3\right)\epsilon^2 + O(\epsilon^3)$$

Use the aforementioned expansions to find x_0, x_1 and x_2.

2. [7pts] If

$$\bar{x}(\epsilon) = x_0 + x_1\epsilon + O(\epsilon^2)$$

is a root of $f(x,\epsilon) = 0$ then general theory implies x_0, x_1 must satisfy

$$\begin{aligned} f(x_0, 0) &= 0 \\ f_x(x_0,0)x_1 + f_\epsilon(x_0,0) &= 0 \end{aligned}$$

Use this approach to find x_0, x_1 where

$$f(x,\epsilon) = \ln(x+\epsilon) - \frac{1}{\sqrt{1+\epsilon x}}$$

3. [7pts] Consider the coupled system

$$\begin{aligned} x^2 - x &= \epsilon(x + 6y) \\ x^2 - y^2 &= \epsilon x^2 \end{aligned}$$

Assume the exact solutions $(\bar{x}(\epsilon), \bar{y}(\epsilon))$ can be expanded as follows:

$$\bar{x}(\epsilon) = x_0 + x_1\epsilon + O(\epsilon^2)$$

$$\bar{y}(\epsilon) = y_0 + y_1\epsilon + O(\epsilon^2)$$

Find x_0, x_1, y_0 and y_1 for the two nonzero roots.