Math 450 (2023) – Homework 3

Due: Friday, October 13, 2023. NAME: ____________________

1. [6pts] Consider the initial value problem:

$$y'' + 4y = \ln(1 + 24\epsilon y^2) \quad , \quad y(0) = 1 \quad , \quad y'(0) = 0$$

where $0 < \epsilon \ll 1$. Find $y_0(t)$ and $y_1(t)$ in the assumed expansion of the solution y:

$$y(t,\epsilon) = y_0(t) + \epsilon y_1(t) + O(\epsilon^2)$$

2. [7pts] Regular perturbation techniques can be applied to approximate the solution of two-point Boundary Value Problems (BVP). Let $y(x)$ be the solution of the nonlinear BVP

$$y' \; y'' = \epsilon x (y')^2 \quad , \quad y(0) = 0 \quad , \quad y(1) = 1$$

where $0 < \epsilon \ll 1$. Notice that the values of $y(x,\epsilon)$ are specified at the boundaries $x = 0$ and $x = 1$.

Find $y_0(x)$ and $y_1(x)$ in the assumed expansion of the solution y:

$$y(x,\epsilon) = y_0(x) + \epsilon y_1(x) + O(\epsilon^2)$$

3. [7pts] Let $y(t,\epsilon)$ be the solution of the initial value problem

$$\begin{aligned} y'' + y &= \epsilon \; y(y')^2 \quad , \quad 0 < \epsilon \ll 1 \\ y(0) &= 0 \quad , \quad y'(0) = 1 \end{aligned}$$

where $(\;)'$ denotes differentiation with respect to t. Assume

$$\begin{aligned} y(t,\epsilon) &= y_0(\tau) + \epsilon y_1(\tau) + O(\epsilon^2) \\ \tau &= \omega(\epsilon) \equiv 1 + \omega_1 \epsilon + O(\epsilon^2) \end{aligned}$$

where $y_k(\tau)$ are $2\pi-$periodic in τ. Use Poincare Lindstedt's method to find $y_0(\tau)$ and the corrected period of the oscillation, i.e., T_0 and T_1 in the exact period (in the original time t):

$$T(\epsilon) = \frac{2\pi}{\omega(\epsilon)} = T_0 + \epsilon T_1 + O(\epsilon^2)$$

You will need to look up appropriate trigonometric identities to complete the problem. You do not need to find $y_1(\tau)$.