Math 451 (2018) – Homework 7 (max=20)

Due: Friday, February 9, 2018. NAME: ____________________

1. [10 pts] Find the extrema of

$$J(y) \equiv \int_0^1 xy(x)\ dx$$

over

$$\mathcal{A} = \left\{ y \in C^2[0,1] : y(0) = 0, y(1) = 0 \right\}$$

subject to the constraint

$$K(y) \equiv \int_0^1 y'(x)^2\ dx = 1$$

2. [5 pts] Consider the motion of a particle in the (x, y)-plane with Lagrangian

$$L = T - V = \frac{1}{2} m \left(\dot{x}^2 + \dot{y}^2 \right) - V(r)$$

where $V(r)$ is the potential energy and T is the kinetic energy. Re-express the Lagrangian in polar coordinates (r, θ), i.e., $L = L(r, \dot{r}, \theta, \dot{\theta})$ where "dot" is a time t derivative. Then, write out the Euler-Lagrange equations. These are the equations of motion for planar motion of a particle under the influence of a radially symmetric force. Don't attempt to solve them.

3. [5 pts] Let Γ be a geodesic on the the graph $z = f(x, y) = y - 2x^2$ where (i) $y = y(x)$ is a function of x on Γ, (ii) $0 < x < 1$ and (iii) the y-values are known at each endpoint. Under these assumptions, the length functional is given by $L(x, y, y')$ the length functional

$$J(y) = \int_0^1 \sqrt{1 + y'(x)^2 + \left(\frac{d}{dx} f(x, y(x)) \right)^2}\ dx = \int_0^1 L(x, y, y')\ dx$$

where $y' \equiv \frac{dy}{dx}$. If you've done things correctly, $L_y = 0$ so that $0 = \frac{d}{dx} L_{y'}$ and $L_{y'} = c$ is constant. This is the first integral for the Euler Lagrange equations defining the geodesic. Explicitly write out this first integral equation $L_{y'} = c$. Do not attempt to solve it.