Math 451 (2024) – Homework 9

(THROUGHOUT USE INTEGRAL TABLES AND ODE SOLVERS AS NEEDED)

Due: Monday April 1, 2024. NAME: _____

1. [10 pts] Find the general solution of the following partial differential equations:

a) $tu_{xx} - 4u_x = 0$ where $u = u(x,t)$ (Hint: $v = u_x$.)

b) $u_{xt} + \frac{1}{x}u_t = \frac{t}{x^2}$ where $u = u(x,t)$.

2. [5 pts] The solution $u(x,t)$ of the initial value problem

$$\begin{aligned} u_t &= Du_{xx} \quad , \qquad x \in \mathbb{R} \quad , \quad t > 0 \\ u(x,0) &= f(x) = e^{-x^2} \end{aligned}$$

is given by the fundamental solution of the heat equation (see notes or text).

a) Find an explicit (simplified) formula for $u(x,t)$ using:

$$\int_{-\infty}^{\infty} e^{-(ax^2+bx+c)}dx = \sqrt{\frac{\pi}{a}}\, e^{(b^2-4ac)/4a}$$

b) Use your formula for the solution $u(x,t)$ to determine the time T at which the peak of $u(x,t)$ is one third its original value of one occurring at $x = 0$, i.e.

$$u(0,T) = \frac{1}{3}f(0)$$