Theorem 0.1 *If there is a pair* (μ^*, x^*) *for which*

$$
\begin{aligned}
f(x^*, \mu^*) &= 0 \quad (4)\\
f_x(x^*, \mu^*) &= 0 \quad (5)\\
f_\mu(x^*, \mu^*) &\neq 0 \quad (6)\\
f_{xx}(x^*, \mu^*) &\neq 0 \quad (7)
\end{aligned}
$$

then $\dot{x} = f(x, \mu)$ *has a saddle-node bifurcation with quadratic tangency at* (μ^*, x^*).

Transcritical (2-branch)

Theorem 0.2 *If there is a pair* (μ^*, x^*) *for which*

$$
\begin{aligned}
f(x^*, \mu^*) &= 0 \quad (8)\\
f_x(x^*, \mu^*) &= 0 \quad (9)\\
f_\mu(x^*, \mu^*) &= 0 \quad (10)\\
f_{x\mu}(x^*, \mu^*) &\neq 0 \quad (11)\\
f_{xx}(x^*, \mu^*) &\neq 0 \quad (12)
\end{aligned}
$$

then $\dot{x} = f(x, \mu)$ *has a (2 branch) transcritical bifurcation at* (μ^*, x^*).

Pitchfork (Quadratic Tangency)

Theorem 0.3 *If there is a pair* (μ^*, x^*) *for which*

$$
\begin{aligned}
f(x^*, \mu^*) &= 0 \quad (13)\\
f_x(x^*, \mu^*) &= 0 \quad (14)\\
f_\mu(x^*, \mu^*) &= 0 \quad (15)\\
f_{xx}(x^*, \mu^*) &= 0 \quad (16)\\
f_{x\mu}(x^*, \mu^*) &\neq 0 \quad (17)\\
f_{xxx}(x^*, \mu^*) &\neq 0 \quad (18)
\end{aligned}
$$

then $\dot{x} = f(x, \mu)$ *has a pitchfork bifurcation with quadratic tangency at* (μ^*, x^*).