# **Math 454 (2018)** Assignment 1

(Due: Wednesday September 19, 2018 in class)

**1.** (5) Rewrite the following scalar differential equation as systems of autonomous differential equations and then state whether the resulting systems are linear or nonlinear.

$$\frac{d^2x}{dt^2} - x\frac{dx}{dt} + x = 1$$

**2.** (5) Find two solutions of the initial value problem

$$\dot{x} \quad = \quad \sqrt{1 - x^2} \quad , \quad x(0) = 1 \tag{1}$$

$$\tag{2}$$

For each of your solutions state the maximum range of $t$ for which they exist.

**3.** (15) For the following differential equations use phase portraits to classify all fixed points (as stable or unstable). If the equilibria is globally stable or globally unstable state so.

a) $\dot{x} = (1 - x)(2 - x)$

b) $\dot{x} = x^2(6 - x)$

**4.** (5) Using Picard iteration find a (infinite) series representation of the solution to

$$\dot{x} = x \quad , \quad x(0) = 1.$$

Does the series converge to the unique solution $x(t) = e^t$?

**5.** (5) Let $x(t)$ be the solution of

$$\dot{x} = x^2 - 1 \quad , \quad x(0) = 1.$$

Compute the first four terms of the Taylor series of $x(t)$ about $t = 0$, i.e.,

$$x(t) = x(0) + x'(0)t + \frac{1}{2}x''(0)t^2 + \frac{1}{3}x'''(0)t^3 + \cdots$$

**6.** (5) Compute the potential function $V(x)$ for the system

$$\dot{x} = r + x - \frac{1}{3}x^3$$

where $r \in \mathbb{R}$ is a parameter. Sketch $V(x)$ if $r \in [0, 2/3)$ labelling the location of fixed points and categorizing them as stable or unstable.