Math 454 (2018) Assignment 2

(Due: Wednesday, October 3, 2018. in class)

1. (5) Assume

$$\dot{x}(t) = f(x(t)) \quad , \quad x(0) = x_0$$

has exactly two fixed points $\bar{x}_1 < \bar{x}_2$ and $f(x)$ is smooth. Thus on the interval $(\bar{x}_1, \bar{x}_2)$ $f(x)$ is of one sign.

Now define a transformation $x \to y$

$$y = \phi(x) \equiv \int_{x_0}^{x} \frac{ds}{f(s)} \quad , \quad \forall x, x_0 \in (\bar{x}_1, \bar{x}_2) \; . \tag{1}$$

a) For $\dot{x} = x^2 + 1$ and $x_0 = 0$ compute $\phi(x)$ and its inverse function $\phi^{-1}(y)$.

b) Prove (for any such $f(x)$) that the inverse $\phi^{-1}(y)$ exists (Hint: consider the sign of $\frac{d\phi}{dx}$).

c) Show that $\dot{y} = g(y)$ for some REALLY simple function $g(y)$ by differentiating $y(t) = \phi(x(t))$ in t in equation (1) above.

2. (25pts) For each of the following scalar ordinary differential equations draw a qualitatively accurate bifurcation diagram (you may use graphical software) labeling all bifurcation points (μ^*, x^*) and the type of bifurcation (SN,TC,PF). State (μ^*, x^*) exactly. Solid=stable, dashed=unstable.

$$\begin{aligned}
a) \quad \dot{x} &= \mu - \frac{x^2}{x^2+1} && (2)\\
b) \quad \dot{x} &= (x-1)(x-\mu) && (3)\\
c) \quad \dot{x} &= (x-1)(\mu - 2x + x^2) && (4)\\
d) \quad \dot{x} &= x(\mu + x - x^2) && (5)\\
e) \quad \dot{x} &= \mu + x - x^3 && (6)
\end{aligned}$$