Math 454 (2018) Assignment 3

(Due: Wednesday, October 17, 2018. in class)

1. (6) For the following three differential equations find ALL the non-hyperbolic fixed points. Do not draw any bifurcation diagrams.

$$
\begin{aligned}
a) \quad \dot{x} &= \mu + x - \ln(1+x) \qquad (1)\\
b) \quad \dot{x} &= \mu - x^2(x^2-4)^2 \qquad (2)\\
c) \quad \dot{x} &= \mu x - e^{\mu x} + \mu \qquad (3)
\end{aligned}
$$

2. (6) For

$$\dot{x} = f(x, \mu) = x(\mu - e^x)$$

identify all nonhyperbolic fixed points that are transcritical bifurcations. Sketch the bifurcation diagrams labelling fixed point stability. Also find the normal form, i.e., the second order Taylor series about $y = x - x^*, \eta = \mu - \mu^*$. This should look something like

$$\dot{y} = ay(b \pm y) + O(3)$$

where a and b possibly depend on some new parameter η.

3. (8) Let

$$\dot{x} = f(x, \mu) = (\mu^2 - 1)(x^2 - 4).$$

Draw the following 4 diagrams in the (μ, x) plane

a) Label only the location of all nonhyperbolic fixed points

b) Label only the location of all isolated fixed points

c) Label only the location of all non-isolated fixed points

d) Label all those fixed points that are stable but not asymptotically stable

4. (10) Consider the following differential equation on S^1:

$$\dot{\theta} = f(\theta, \mu) = \cos\theta + \mu$$

a) Draw a phase portrait indicating the flow direction and stability of all fixed points when $\mu = 1/2$

b) For $\mu = 3$ compute the period of the periodic orbit. You may use Integral tables or software to compute the difficult integral involved.