Homework 5 (Math 454) **Due: Wednesday, November 28, 2018.**

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1. [5] Find constants α and β such that

$$E = \frac{x^4}{a^4} + \frac{y^4}{b^4}$$

is a first integral of the system:

$$\begin{aligned} \dot{x} &= \alpha y^3 \\ \dot{y} &= \beta x^3 \end{aligned}$$

if $a^4\beta + \alpha b^4 = 0$. Compute the Jacobian about the origin. Does the linearized system indicate the system has a center? Lastly, sketch the trajectories of the system when E is a first integral.

2. [10] For each of the following Hamiltonian systems, find the Hamiltonian $H(x, y)$. Use the level sets of H to draw an accurate phase portrait for the system indicating flow direction.

a) $\begin{aligned} \dot{x} &= -x^3 \\ \dot{y} &= 3x^2 y \end{aligned}$ **b)** $\begin{aligned} \dot{x} &= 1 - y \\ \dot{y} &= x - 1 \end{aligned}$

3. [5] Consider the following planar system (with initial condition)

$$\dot{x} = -x \qquad x(0) = x_0 \tag{1}$$

$$\dot{y} = x(1 - y) \qquad y(0) = y_0 \tag{2}$$

Find the flow function $\phi(t, x_0, y_0)$. For $x_0 \neq 0$ what does $\phi(t, x_0, y_0)$ approach as $t \to \infty$?

4. [10] Consider the system

$$\begin{aligned} \dot{x} &= y(1 - x^2) \\ \dot{y} &= 1 - y^2 \end{aligned}$$

i) Is the system Hamiltonian?

ii) Is the system reversible?

iii) Find all the fixed points and deduce which (if any) of the fixed points are hyperbolic.

iv) Sketch the phase portrait (indicating flow direction).