Math 455 Final
Friday, April 28, 2017. Name: ____________________

Instructions: All work must be shown to receive full credit. You may use your notes, posted notes and textbook but may not talk to any fellow students. I will answer clarifying questions about concepts but not about the details of a specific problem.

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1. [40] The following questions concern the continuous map $f : \mathbb{R} \to \mathbb{R}$.

$$f(x) = \begin{cases} x+1 & , \quad x \le 1 \\ 4-2x & , \quad x > 1 \end{cases} \tag{1}$$

a) Draw/plot an accurate graph of $f(x)$ with $y = x$ indicating the location of the fixed point.
b) Determine the coordinate and stability of the fixed point $\bar{x}$ of f.
c) The interval $[0, 2]$ is partitioned into two disjoint sets $L = [0, 1)$ and $R = [1, 2]$. Draw the transition graph of f for this cover.

d) For this cover, find the first six symbols in the itinerary $I(\gamma(0))$ of the orbit $\gamma(0)$.
e) Find <u>all</u> those <u>positive</u> $x_0 \neq \bar{x}$ such that $\gamma(x_0)$ eventually reaches $\bar{x}$ in three iterates, i.e.,

$$f^3(x_0) = \bar{x}$$

and $\gamma(x_0) = \{x_0, x_1, x_2, \bar{x}, \bar{x}, \bar{x}, \ldots\}$ where x_0, x_1, x_2 are all positive. <u>Do not</u> compute or plot $f^3(x)$. To get you started, find x_2 such that $f(x_2) = \bar{x}$.

2. [20] Let $f(x) = x(a - x)$ define a scalar map on $\mathbb{R}$. Period 2 orbits correspond to certain roots of

$$Q(x) = f^2(x) - x = -x(x - a + 1)(x^2 - (a + 1)x + (a + 1)) \ .$$

a) For what <u>positive</u> values of a does $f(x)$ have a minimal period 2 orbit?
b) If $a = \frac{7}{2}$, is the period two orbit stable or unstable?

3. [15] Find the formula for the <u>quadratic</u> map $f(x)$ that has the origin as a fixed point and the period two orbit:

$$\gamma(p) = \{1, 2, 1, 2, 1, 2, \ldots\}$$

4. [10] Let $f : [0, 1] \to [0, 1]$ and $[0, 1]$ have a cover $[0, 1] = L \cup R$ where L and R are disjoint sets. The orbit $\gamma(x_0) = \{x_0, x_1, x_2, \ldots\}$ has the itinerary

$$I(\gamma(x_0)) = \{L, L, R, R, \overline{L, L, R, R} \ldots\}$$

What is the itinerary of $I(\gamma(x_3))$?

5. [15] The map $f(x) = x^2 + 1$ is conjugate to $g(x) = 3x^2 - 2x + 1$ via the homeomorphism $H(x) = Ax + B$ for some constants A, B. Find A and B, i.e., so that $H(f(x)) = g(H(x))$ for all x.