Math 454 (2016)

Assignment 3 (Due: October 14, 2016 in class)

1. [18pt] Consider the following differential equations where $x \in \mathbb{R}, \mu \in \mathbb{R}$:

$$\begin{aligned} \dot{x} &= f(x,\mu) = \mu + x - ln(1+x) && (1) \\ \dot{x} &= f(x,\mu) = \mu - x^2(x^2-4)^2 && (2) \\ \dot{x} &= f(x,\mu) = \mu x - e^{\mu x} + \mu && (3) \end{aligned}$$

For (1)-(3)

i) identify all nonhyperbolic fixed points (μ^*, x^*)

ii) determine which are saddle nodes by verifying $f_\mu \neq 0, f_{xx} \neq 0$

iii) Find the second order Taylor series approximation of f in (x,μ) about each (μ^*, x^*)

iv) Find the "normal form" of all saddle nodes in (η, y) where $y = x - x^*, \eta = \mu - \mu^*$. This should look like $\dot{y} = \eta \pm ay^2 + O(3)$ where a possibly depends on the new parameter η.

2. [18pt] Consider the following differential equations where $x \in \mathbb{R}, \mu \in \mathbb{R}$:

$$\begin{aligned} \dot{x} &= f(x,\mu) = \mu x - ln(x+1) && (4) \\ \dot{x} &= f(x,\mu) = x(\mu - e^x) && (5) \\ \dot{x} &= f(x,\mu) = (\mu - x)(2\mu - x) && (6) \end{aligned}$$

For (4)-(6)

i) identify all nonhyperbolic fixed points (μ^*, x^*)

ii) determine which are transcritical bifurcations by verifying $f_\mu = 0, f_{xx} \neq 0, f_{x\mu} \neq 0$

iii) Find the second order Taylor series approximation of f in (x,μ) about each (μ^*, x^*)

iv) Find the "normal form" for (4)-(5) of all transcritical bifurcations in (η, y) where $y = x - x^*, \eta = \mu - \mu^*$. This should look like $\dot{y} = ay(b \pm y) + O(3)$ where a, b possibly depend on the parameter η.

3. [9] Define the system

$$\dot{x} = f(x,\mu,\lambda) = \lambda x + \mu x^2 - x^3 = xg(x,\mu,\lambda) \qquad (7)$$

where λ, μ are parameters. When $\lambda = 0$ the system has a pitchfork in μ. Find the locus of nonhyperbolic points Γ, i.e. the curve in the (μ, λ) plane defined by:

$$\begin{aligned} f(x,\mu,\lambda) &= 0 && (8) \\ f_x(x,\mu,\lambda) &= 0 && (9) \end{aligned}$$

Plot Γ and deduce (shade in) the region in the (μ, λ) plane where f has three roots. Write out explicit formulae for all three real roots when they exist.