Homework 5 (Math 454) Due: Wednesday, November 16, 2016.

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1. [18] For the following systems,

i) Sketch the nullclines and indicate the direction of flow across them.

ii) Find all fixed points and classify them using linear stability analysis (saddle, stable node, center, etc.).

iii) For saddles, make sure your trajectories are tangent to the associated eigenvectors defining E^s and E^u.

iv) Lastly, draw a "plausible" phase portrait, i.e, consistent with the information previously obtained.

a)

$$\dot{x} = y \tag{1}$$
$$\dot{y} = x(x^2 - 1) \tag{2}$$

b)

$$\dot{x} = x^2 - y \tag{3}$$
$$\dot{y} = x - y \tag{4}$$

2. [6] Find constants α and β such that

$$E = \frac{x^4}{a^4} + \frac{y^4}{b^4}$$

is a first integral of the system:

$$\dot{x} = \alpha y^3$$
$$\dot{y} = \beta x^3$$

if $a^4\beta + \alpha b^4 = 0$. Compute the Jacobian about the origin. Does the linearized system indicate the system has a center? Lastly, sketch the trajectories of the system when E is a first integral.

3. [10] For each of the following Hamiltonian systems, find the Hamiltonian $H(x, y)$. Use the level sets of H to draw an accurate phase portrait for the system indicating flow direction.

a) $\dot{x} = -x^3$, $\dot{y} = 3x^2y$

b) $\dot{x} = 1 - y$, $\dot{y} = x - 1$

4. [6] Consider the following planar system (with initial condition)

$$\dot{x} = -x \qquad x(0) = x_0 \tag{5}$$
$$\dot{y} = x(1 - y) \qquad y(0) = y_0 \tag{6}$$

Find the flow function $\phi(t, x_0, y_0)$. For $x_0 \neq 0$ what does $\phi(t, x_0, y_0)$ approach as $t \to \infty$?