Homework 6 (Math 454) Due: Wednesday, November 30, 2016.

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1. [10] This exercise is meant to show that different systems can have the same first integrals. A first integral is given by

$$E = x^2 + x^2y^2 + y^2$$

i) You are given the origin is an isolated fixed point of $\dot{x} = f(x)$. Prove E has a local min or max at X and hence, by a theorem, is a nonlinear center (compute the Discriminant).

ii) If E is the Hamiltonian $H(x, y)$ find the vector field, i.e., f_1 and f_2 in

$$\begin{aligned} \dot{x} &= f_1(x, y) \\ \dot{y} &= f_2(x, y) \end{aligned}$$

iii) Show that E is also a first integral of

$$\begin{aligned} \dot{x} &= F_1(x, y) = y \\ \dot{y} &= F_2(x, y) = -\frac{x(y^2+1)}{x^2+1} \end{aligned}$$

iv) Use software to plot the trajectories of the system in ii) and the the system in iii) indicating the direction of the flow.

2. [8] Consider the system

$$\begin{aligned} \dot{x} &= y(1 - x^2) \\ \dot{y} &= 1 - y^2 \end{aligned}$$

i) Is the system Hamiltonian?

ii) Is the system reversible?

iii) Find all the fixed points and deduce which (if any) of the fixed points are hyperbolic.

iv) Sketch the phase portrait (indicating flow direction).