Linear Stability

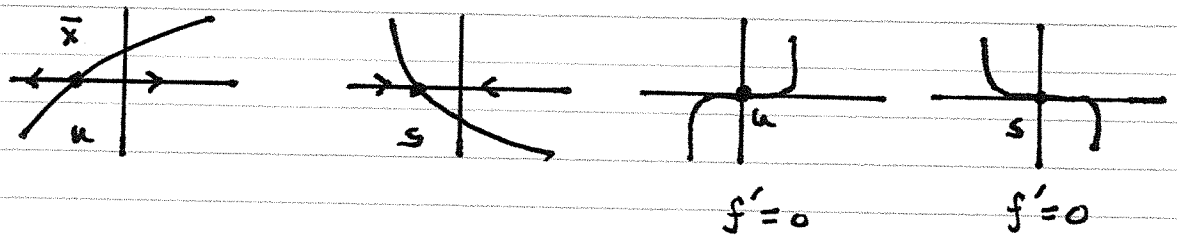
Defn: Let $f: \mathbb{R} \to \mathbb{R}$ be continvously differentiable and $\bar{x}$ be a fixed point of

$$\dot{x} = f(x)$$

Then

(i) $f'(\bar{x}) > 0 \Rightarrow \bar{x}$ unstable
(ii) $f'(\bar{x}) < 0 \Rightarrow \bar{x}$ linearly stable (and stable)
(iii) $f'(\bar{x}) = 0 \Rightarrow$ inconclusive stability

Pf (graphical)



EXAMPLE $\dot{x} = x^3 - 5x^2 + 8x - 4$ has two fixed points

$$\bar{x}_1 = 1 \qquad \bar{x}_2 = 2$$

Here $f'(x) = 3x^2 - 10x + 8$

$f'(1) = 1 > 0 \Rightarrow \bar{x}_1 = 1$ unstable

$f'(2) = 0 \Rightarrow$ inconclusive

what is stability of $\bar{x}_2$ though?