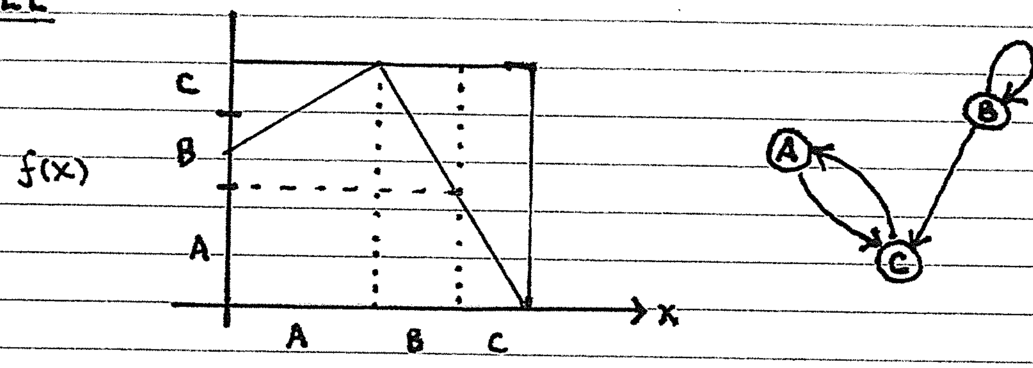
Defn: An <u>allowable path</u> in a transition graph is a path through the vertices following the arrows.

EXAMPLE



$I = \{A, C, A, C, \ldots\}$ allowable

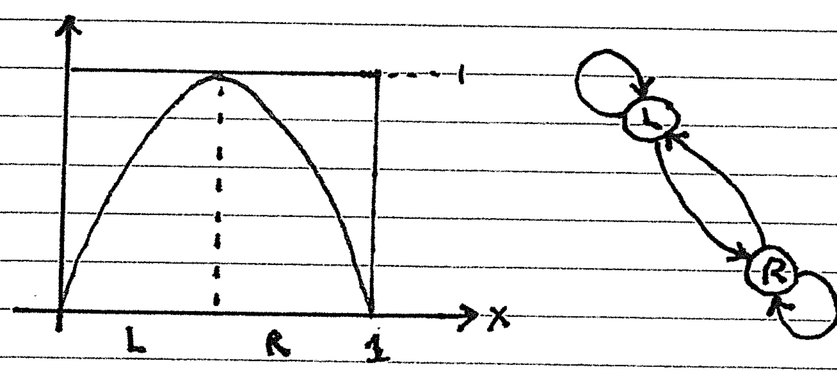
$I = \{B, B, C, A, \ldots\}$ allowable

$I = \{B, B, A, C, \ldots\}$ not allowable

EXAMPLE

~~f(x)~~

Logistic



Because of the transition graph <u>all</u> itineraries are possible.

Shared Itineraries

Let $f: \mathbb{R} \to \mathbb{R}$ and $f(A) = A$ onto. Define an N-symbol cover

$$A = \bigcup_{j=1}^{N} A_j \qquad A_i \cap A_j = \phi \qquad i \neq j$$

with associated orbits and itineraries

$$\gamma(x_0) = \{x_0, x_1, \ldots\}$$

$$I(\gamma) = \{S_0, S_1, S_2, \ldots\} \qquad \text{(allowable)}$$

where $S_j = A_j$ if $x_j \in A_j$.

A natural question to ask is which orbits share the same itinerary or at least the first n-symbols. Toward this end we define the following list (with NO commas)

$$\boxed{S_0 S_1 \cdots S_n = \{x_0 \in A : x_j \in A_j \quad \forall j = 0, \ldots n\}}$$

For example, $f(x) = 4x(1-x)$ we found

$$LL = [0, a_1)$$

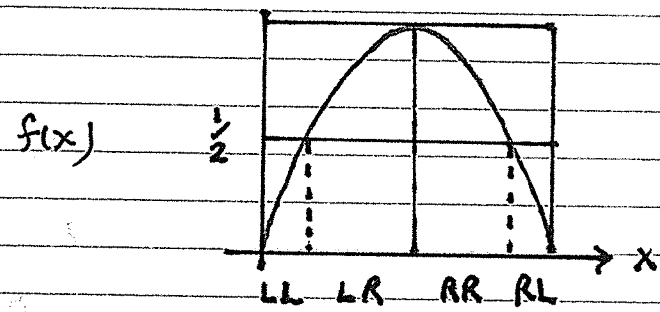
We examine two maps in more detail.

Logistic Map itineraries $\quad f(x) = 4x(1-x)$

For the standard $L = [0, \frac{1}{2})$, $R = [\frac{1}{2}, 1]$ covering we define the intervals I_{ij} :

$$I_{ij} = \{ x_0 : \gamma(x_0) \text{ share same first } j \text{ symbols in } I(\gamma(x_0)) \}$$

For the first iterate map

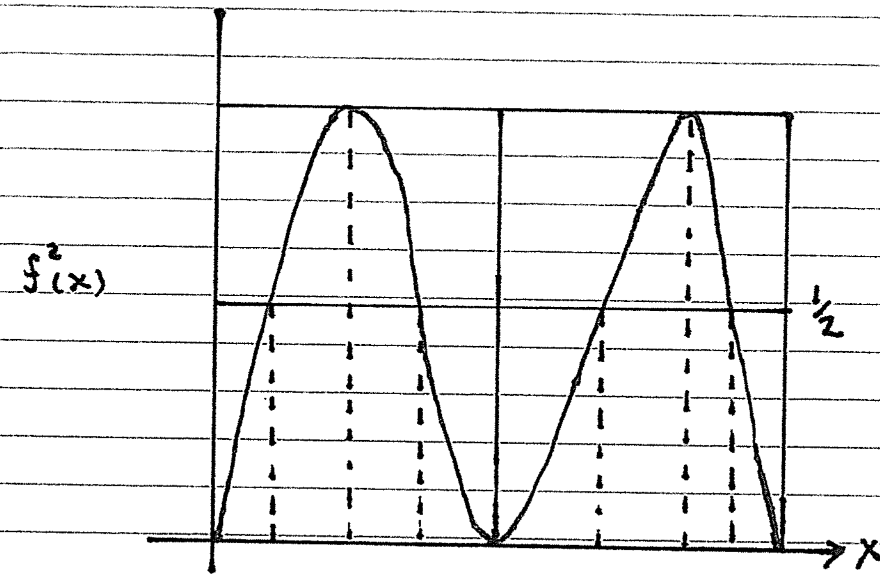


From this we deduce intervals I_{i2}

$I(\gamma(x_0))$	I_{i2}		i
$\{L, L, \ldots\}$	$[0, a_1)$	LL	1
$\{L, R, \ldots\}$	$[a_1, \frac{1}{2}]$	LR	2
$\{R, R, \ldots\}$	$[\frac{1}{2}, a_2]$	RR	3
$\{R, L, \ldots\}$	$(a_2, 1]$	RL	4

The index $i = 1, 2, 3, 4$ index is for all intervals when $j = 2$.

For the second iterate map $\quad j=3 \quad i=2^3$



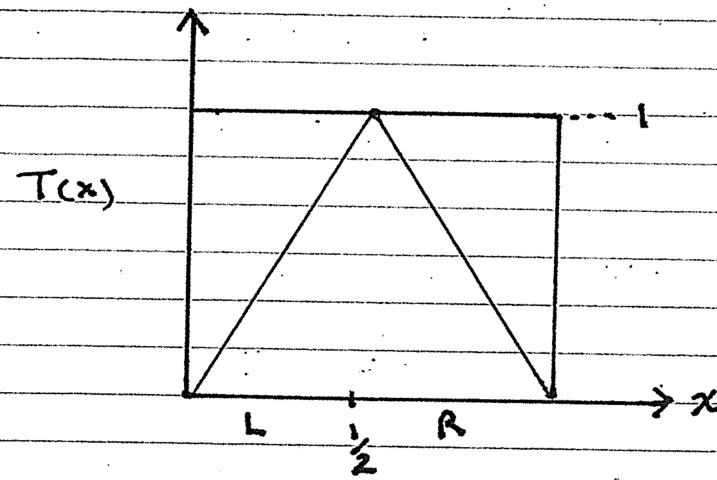
LLL, LLR, LRR, LRL, RRL, RRR, RLR, RLL

Interval spacing

In "Chaos: Introduction to Dynamical Systems" by Alligood, Sauer, Yorke they prove:

$$|I_{ij}| \leq \frac{\pi}{2^j} \qquad \forall i = 1, 2 \ldots 2^j$$

EXAMPLE $T : [0,1] \to [0,1]$ **Tent Map**

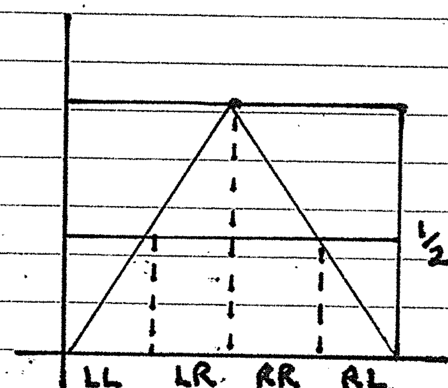


Tent map at left also maps $[0,1]$ onto.

It is piecewise defined below.

$$T(x) = \begin{cases} 2x & x \in L \\ 2-2x & x \in R \end{cases}$$

Finding the pre-images of $\frac{1}{2}$ we can determine shared itineraries.



$$LL = \{x_0 : x_0 \in L, x_1 \in L\} = [0, \tfrac{1}{4}]$$

Similarly one can show

$$LLR = \{x_0 : x_0 \in L, x_1 \in L, x_2 \in R\} = [\tfrac{1}{8}, \tfrac{1}{4}]$$