MATH 472: Final - Take Home (max=60)

Due: April 29, 2016. Name:_____

Instructions: Staple this sheet to your solutions.

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1. [10] Find the first three terms of the Taylor series of

$$f(z) = \frac{z}{2i - z}$$

about $z = i$ and then sketch the region of convergence. You may use Taylor's Theorem or let $w = z - i$ and expand f as a (geometric) series in w.

2. [15] Compute the residue $Resf(z_0)$ of f at the indicated values of z_0 for both of the following functions:

$$f(z) = \frac{\log(3+z)}{(z+2)^3} \quad , \quad z_0 = -2$$

$$f(z) = \frac{z^2}{(z+1)(z-i)^2} \quad , \quad z_0 = -1, i$$

3. [10] Find the Laurent series expansion of

$$f(z) = \frac{2}{(z+2)(z+4)}$$

valid for $2 < |z| < 4$. The terms of the series have the form $a_n z^n$. You need only state the terms with $n = -2, -1, 0, 1, 2$.

4. [12] Use Residue theory to evaluate the integral

$$I = \int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2}\, dx$$

You must show all work including the limit calculations for $\int_{C_R} f(z)\, dz$ as $R \to \infty$.

5. [13] Use Residue theory to evaluate the integral

$$I = \int_{0}^{\infty} \frac{1}{x^4+1}\, dx$$

You must show all work including the limit calculations for $\int_{C_R} f(z)\, dz$ as $R \to \infty$.