Math 472: Homework 1
Due: January 29, 2016.

1. [18] Below are conditions which define a set $S \subset \mathbb{C}$. Sketch each set and state if it is opened, closed, and/or bounded or unbounded.

a) $|z - 2 + i| \leq 1$.

b) $\mathbb{R}e(z) = 1$.

c) $N_r(i) = \{z : |z - i| < r\}$ where $r = 1$.

d) $|2z + 3| \geq 4$

e) $Im(z) > 1$

f) $0 \leq Arg(z) < \frac{\pi}{4}$, $1 < |z| < 4$

2. [24] Computations:

a) Compute $\left|\frac{1+2i}{3-4i}\right|$

b) Sketch all solutions of $z^5 = i$ on the circle $|z| = 1$ labelling all relevant angles.

c) Locate all solutions of $z^3 = 8 - 8\sqrt{3}i$ on a circle of appropriate radius carefully labelling all relevant angles as in b).

d) Compute $\sqrt{2 - 3i}$ in <u>non-polar</u> form.

e) Factor
$$P(z) = 2i\ z^2 + (3i - 7)\ z + (1 - 5i) \quad .$$

f) Using Euler's formula and the <u>non-polar</u> square root formulae, find the value of $\cos(\pi/12)$ expressed as a radical.

3. [8] Miscellaneous:

a) Use DeMoivre's formula to show

$$\cos 3\theta = \cos^3 \theta - 3\cos\theta \sin^2 \theta$$

b) For $P(z) = z^3 + 3z^2 + 2z - 6$, find a real number M such that

$$|P(z)| < M$$

for z on the unit circle $|z-1| = 1$. Hint: if $z = w+1$ then find an upper bound of $|P(z)|$ on $|w| = 1$.