Math 472: Homework 2
Due: Wednesday, February 17, 2016.

1. [4] Let $f(z) = e^{i\pi/4}z$ and S be the unit square (region)

$$S = \{(x, y) : 0 \leq x, y \leq 1\}$$

Draw an accurate sketch of S and the image $f(S)$.
2. [4] Let $f(z) = \bar{z} + i$ and S be the semicircular region

$$S = \{(x, y) : x^2 + y^2 \leq 1, y \geq 0\}$$

Draw an accurate sketch of S and the image $f(S)$.
3. [4] Let $f(z) = z^2 - 1$ and S be the unit circle. Draw an accurate sketch of S and the image $f(S)$. For this, set $z = e^{i\theta}$ and consider the curve described by (u, v) in $f(z) = u(\theta) + iv(\theta)$.
4. [4] Find $u(x, y)$ and $v(x, y)$ if

$$f(z) = \frac{\bar{z}^2}{z} = u(x, y) + iv(x, y)$$

5. [4] Find $u(r, \theta)$ and $v(r, \theta)$ if

$$f(z) = z + \frac{1}{z} = u(r, \theta) + iu(r, \theta) \qquad , \qquad z = re^{i\theta}$$

6. [4] Given

$$f(z) = (x^2 - y^2 - 2y) + i(2x - 2xy)$$

find a formula (simplified) for f in terms of z and $\bar{z}$.
7. [4] Let $f(z) = \sqrt{z}$ be the Principal square root defined by

$$f(z) = \sqrt{r}e^{i\theta/2}, \theta = Arg(z)$$

Use the definition of the derivative to prove

$$f'(z) = \frac{1}{2\sqrt{z}}$$

and state for what z it is not differentiable and why.

8. [4] Show that for all $z \neq 0$, the derivative of $f(z) = z + 2\bar{z}$ does not exist. Give a coherent explanation using the limit definition of $f'(z)$.

9. [4] Show there is no analytic function $f(z)$ with

$$Im(f(z)) = x^2 y - x$$

10. [4] (Errata) Find an analytic function $f(z)$ having the real part:

$$Re(f(z)) = u(r, \theta) = r\cos\theta + \frac{\cos\theta}{r} \qquad , \qquad z = re^{i\theta}$$

Use the polar form of the Cauchy Riemann equations to find the imaginary part $v(r, \theta)$ - to within an additive constant. Then convert back to z.