1. [4] Let \( f(z) = e^{i\pi/4}z \) and \( S \) be the unit square (region)
\[
S = \{(x, y) : 0 \leq x, y \leq 1\}
\]
Draw an accurate sketch of \( S \) and the image \( f(S) \).

2. [4] Let \( f(z) = \bar{z} + i \) and \( S \) be the semicircular region
\[
S = \{(x, y) : x^2 + y^2 \leq 1, y \geq 0\}
\]
Draw an accurate sketch of \( S \) and the image \( f(S) \).

3. [4] Let \( f(z) = z^2 - 1 \) and \( S \) be the unit circle. Draw an accurate
   sketch of \( S \) and the image \( f(S) \). For this, set \( z = e^{i\theta} \) and consider
   the curve described by \((u, v)\) in \( f(z) = u(\theta) + iv(\theta) \).

4. [4] Find \( u(x, y) \) and \( v(x, y) \) if
\[
f(z) = \frac{\bar{z}^2}{z} = u(x, y) + iv(x, y)
\]

5. [4] Find \( u(r, \theta) \) and \( v(r, \theta) \) if
\[
f(z) = z + \frac{1}{z} = u(r, \theta) + iu(r, \theta) \quad , \quad z = re^{i\theta}
\]

\[
f(z) = (x^2 - y^2 - 2y) + i(2x - 2xy)
\]
find a formula (simplified) for \( f \) in terms of \( z \) and \( \bar{z} \).

7. [4] Let \( f(z) = \sqrt{z} \) be the Principal square root defined by
\[
f(z) = \sqrt{r}e^{i\theta/2}, \quad \theta = Arg(z)
\]
Use the definition of the derivative to prove
\[
f'(z) = \frac{1}{2\sqrt{z}}
\]
and state for what \( z \) it is not differentiable and why.
8. [4] Show that for all \( z \neq 0 \), the derivative of \( f(z) = z + 2\bar{z} \) does not exist. Give a coherent explanation using the limit definition of \( f'(z) \).

9. [4] Show there is no analytic function \( f(z) \) with

\[ \text{Im}(f(z)) = x^2y - x \]

10. [4] (Errata) Find an analytic function \( f(z) \) having the real part:

\[ \text{Re}(f(z)) = u(r, \theta) = r \cos \theta + \frac{\cos \theta}{r}, \quad z = re^{i\theta} \]

Use the polar form of the Cauchy Riemann equations to find the imaginary part \( v(r, \theta) \) - to within an additive constant. Then convert back to \( z \).