1. [4] Find all values of $\tan^{-1}(1+i)$ where

$$\tan^{-1} z = \frac{i}{2} \log \frac{i + z}{i - z}$$

2. [6] For both $f(z) \equiv e^{z^3}$ and $g(z) \equiv \bar{z}$ find all $z$ for which
   a) they are analytic
   b) their real part is harmonic
   c) their real and imaginary parts are harmonic conjugates

3. [4] Find all values $z$ such that

$$e^z = -2$$

4. [4] Compute $\text{Log}(-ei)$ where $\text{Log}(z)$ is the Principal branch.

5. [4] For $\log(z)$ defined with $\arg(z) \in [\alpha, \alpha + 2\pi)$, it can be shown in cartesian coordinates

$$\log z = \frac{1}{2} \ln(x^2 + y^2) + i \tan^{-1}\left(\frac{y}{x}\right)$$

Recall $f'(z) = u_x + iv_x$ for any analytic function. Use these facts to prove

$$\frac{d}{dz} \log z = \frac{1}{z}$$

6. [4] For the following $z = 1+i, z_2 = 1-i, z_3 = -1-i$. Furthermore, $z^c$ is the principal branch exponent. Show:

$$(z_1z_2)^i = z_1^iz_2^i$$

$$(z_2z_3)^i = z_2^iz_3^i e^{-2\pi}$$

For each case, write out the values of $(z_mz_n)^i, z_m^i, z_n^i$ and $z_1^iz_2^i$
7. [4] Prove \( \cos(2z) = \cos^2 z - \sin^2 z \).

8. [4] Prove \( |\sin z| \geq |\sin x| \). For this you need:

\[
\sin z = \sin x \cosh y + i \cos x \sinh y
\]

This is a very short proof by considering \( |\sin z|^2 \). Write it out precisely explaining each step.

9. [6] For the following functions \( f(z) \), find the branch cuts and all other places where \( f(z) \) fails to be analytic.

\[
f(z) = \frac{\text{Log}(iz + 1)}{z - 2}
\]

\[
f(z) = \frac{1}{i} \frac{\text{Log}(z^2 - 1)}{z^2 + 9}
\]