Math 472: Homework 3
Due: Wednesday, March 2, 2016.
Midterm 1 will be on Wednesday, March 9.

1. [4] Find all values of $\tan^{-1}(1+i)$ where

$$\tan^{-1} z = \frac{i}{2} \log \frac{i+z}{i-z}$$

2. [6] For both $f(z) \equiv e^{z^3}$ and $g(z) \equiv \bar{z}$ find all z for which

a) they are analytic

b) their real part is harmonic

c) their real and imaginary parts are harmonic conjugates

3. [4] Find all values z such that

$$e^z = -2$$

4. [4] Compute $Log(-ei)$ where $Log(z)$ is the Principal branch.

5. [4] For $log(z)$ defined with $arg(z) \in [\alpha, \alpha + 2\pi)$, it can be shown in cartesian coordinates

$$\log z = \frac{1}{2} \ln(x^2 + y^2) + i \tan^{-1}\left(\frac{y}{x}\right)$$

Recall $f'(z) = u_x + iv_x$ for any analytic function. Use these facts to prove

$$\frac{d}{dz} \log z = \frac{1}{z}$$

6. [4] For the following $z = 1+i, z_2 = 1-i, z_3 = -1-i$. Furthermore, z^c is the prinicipal branch exponent. Show:

$$\begin{aligned} (z_1 z_2)^i &= z_1^i z_2^i \\ (z_2 z_3)^i &= z_2^i z_3^i e^{-2\pi} \end{aligned}$$

For each case, write out the values of $(z_m z_n)^i, z_m^i, z_n^i$ and $z_1^i z_2^i$

7. [4] Prove $\cos(2z) = \cos^2 z - \sin^2 z$.

8. [4] Prove $|\sin z| \geq |\sin x|$. For this you need:

$$\sin z = \sin x \cosh y + i \cos x \sinh y$$

This is a **very** short proof by considering $|\sin z|^2$. Write it out precisely explaining each step.

9. [6] For the following functions $f(z)$, find the branch cuts and all other places where $f(z)$ fails to be analytic.

$$f(z) = \frac{Log(iz+1)}{z-2}$$

$$f(z) = \frac{1}{i}\frac{Log(z^2-1)}{z^2+9}$$