Math 472: Homework 4
Due: Friday, April 1, 2016.

Instructions: In every problem sketch the integration curve and indicate the location of all integrand singularities (with the exception of **1.** which is not analytic anywhere).

1. [5] Evaluate

$$I = \int_C \frac{\bar{z}}{z}\, dz$$

where C is the straight line from $z_1 = 1$ to $z_2 = 3i$. Note that C is given by:

$$z(t) = (1-t)z_1 + tz_2 \qquad , \quad t \in [0,1]$$

2. [5] Let C be the unit semicircle $z = e^{i\theta}$ that crosses the cut of Principal $Log(z)$. The ranges of θ are $\theta \in [\frac{\pi}{2}, \pi^-]$ and $\theta \in [-\pi^+, -\frac{\pi}{2}]$ Evaluate

$$I = \int_C Log(z)\; dz$$

directly using the parametrization $z = e^{i\theta}$. Remember to split the integral into two parts.

3. [5] Let

$$f(z) \equiv \frac{z}{\sqrt{z^2+1}}$$

Draw the cut Γ of $f(z)$ and any curve C (not crossing Γ) from $z_1 = 2+i$ to $z_2 = -2$. Then use an antiderivative of $f(z)$ to evaluate

$$I = \int_C f(z)\; dz$$

4. [5] Evaluate

$$I = \oint_C \frac{e^z}{z-2}\; dz$$

where (a) C is the circle $|z| = 3$ and (b) C is the circle $|z| = 1$.

5. [5] Evaluate

$$I_n \equiv \oint_{|z|=1} \frac{\sin 2z}{z^n}\, dz$$

for $n = 0, 1, 2, 3, 4$.

6. [5] Use partial fraction expansions to evaluate

$$\oint_{|2z|=5} \frac{2z+1}{z^2+3z+2}\, dz$$

7. [5] Prove the real integral

$$\int_0^{\pi} e^{\cos\theta} \cos(\sin\theta)\, d\theta = \pi$$

by considering the real/imaginary part of the complex integral

$$\oint_{|z|=1} \frac{e^z}{z}\, dz$$

8. [5] Evaluate

$$\oint_C \frac{dz}{(z^2+1)(z^2+4)}$$

where C is the upper $(Im(z) > 0)$ closed semicircle of radius 3.