Math 472: Homework 5
Due: Monday April 18, 2016.

1. [4] While proving the Fundamental Theorem of Algebra we showed

$$\left|\frac{1}{P(z)}\right| < \frac{2}{|a_n|R^n} \quad , \quad |z| \geq R$$

for any polynomial

$$P(z) = a_0 + a_1 z + \cdots + a_{n-1}z^{n-1} + a_n z^n$$

Use this to find an upper bound on the integral

$$\int_{C_R} \frac{1}{P(z)}\, dz$$

where C_R is a circle of radius R centered at the origin and $P(z)$ is a polynomial of degree $N = 0, 1, 2...n$. For which N does the integral vanish as $R \to \infty$.

2. [4] A nonconstant function $F(z)$ is periodic on a rectangle in the sense that there are real numbers $a, b > 0$ such that

$$F(z+a) = F(z) \qquad F(z+ib) = F(z) \qquad \forall z$$

Prove $F(z)$ can't be analytic on the rectangle $\mathcal{R}$ given by $0 \leq x \leq a$, $0 \leq y \leq b$ using Liouville's Theorem. Let M as the maximum of $|F(z)|$ on the rectangle and carefully write out your answer (a sentence or two).

3. [4] Use the geometric series to find the Taylor Series of

$$f(z) = \frac{2}{2 + i - z}$$

centered at $z_0 = i$. What is the radius of convergence R? Draw the region of convergence.

4. [12] For each of the following find the first three nonzero terms in the Taylor Series about the indicated points. In each case determine the region of convergence.

$$\begin{aligned} f(z) &= \cos z \quad , \quad z_0 = \pi/2 \\ f(z) &= \frac{1}{z+1} \quad , \quad z_0 = 1 \\ f(z) &= \frac{1}{(z-1)(z-4)} \quad , \quad z_0 = 2 \\ f(z) &= z^2 + z + 1 \quad , \quad z_0 = 2 \end{aligned}$$

5. [5] Use Taylor's Theorem (no shortcuts) to find the first four nonzero terms of the Taylor Series of the Principal Branch

$$f(z) = Log(3 - iz)$$

about $z_0 = 2i$. Derive the cut location and sketch the region of convergence.

6. [6] Find Laurent expansions (first 4 terms) of

$$f(z) = \frac{1}{z(z-2)}$$

valid for i) $0 < |z| < 2$ and ii) $0 < |z-1| < 1$.

7. [5] Expand

$$f(z) = \frac{z}{(z-1)(z-2)}$$

in a Laurent series valid on the annular region $1 < |z| < 2$. First do a partial fraction expansion and include only the terms $a_n z^n$ for $n = -2, -1, 0, 1, 2$.