Field Axioms

In Mathematics a Field $\mathcal{F}$ is any set endowed with addition and multiplication operations that satisfy the following ten axioms:

For Addition:

$$\begin{array}{llrcll}
\text{A1)} & & a+b & = & b+a & \\
\text{A2)} & & (a+b)+c & = & a+(b+c) & \\
\text{A3)} & \exists 0 \in \mathcal{F} \text{ such that} & a+0 & = & a & \\
\text{A4)} & \exists b \in \mathcal{F} \text{ such that} & a+b & = & 0 & \Rightarrow b = -a
\end{array}$$

For Muliplication:

$$\begin{array}{llrcll}
\text{M1)} & & ab & = & ba & \\
\text{M2)} & & (ab)c & = & a(bc) & \\
\text{M3)} & \exists 1 \in \mathcal{F} \text{ such that} & 1a & = & a & \\
\text{M4)} & \forall a \neq 0, \exists b \text{ such that} & ab & = & 1 & \Rightarrow b = a^{-1} = \frac{1}{a}
\end{array}$$

Distributive Laws:

$$\begin{array}{lrcl}
\text{D1)} & a(b+c) & = & ab+ac \\
\text{D2)} & (a+b)c & = & ac+bc
\end{array}$$

The Real numbers $\mathbb{R}$ are a field. So is $\mathbb{R}^2$ if multiplication is defined the right way. The Complex plane (number system) is $\mathbb{R}^2$ with this definition of multiplication.

<u>Notes on notation:</u>

$\exists$ mean "there exists"

$\forall$ mean "for all"

$\ni$ mean "such that"