Math 560 Homework 1

Due: Sept 22, 2016

1. [5pt] Define the (ordered) set $T := \{1, x, x^2\}$ in $L^2[0,1]$ with the weighted inner product

$$< f, g >= \int_0^1 x \ f(x)g(x) \ dx \tag{1}$$

Use the Gram Schmidt procedure to find an othogonal basis for $span(T)$.

2. [5pt] Let $S = \mathbb{R}^3$ and $f = (4, 1, -1)$. Find the coordinate $\vec{\alpha}$ of f relative to $T = \{\phi_1, \phi_2, \phi_3\}$ where $\phi_1 = (1,1,0)^T, \phi_2 = (0,1,1)^T, \phi_3 = (1,0,1)^T$.

3. [5pt] Orthogonally diagonalize

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

4. [5pt] Let $P(x)$ be the projection of $x \in \mathbb{R}^3$ onto $T = span\{\phi_1, \phi_2\}$ where $\phi_1 = (1, 0, -1)^T, \phi_2 = (1, 1, 1)^T$. Find the matrix A such that $P(x) = Ax$. Then find all the eigenvalues of A. Is A invertible? Note ϕ_k aren't normalized.

5. [7pt] Consider the eigenvalue problem

$$A(\varepsilon)x(\varepsilon) = \lambda(\varepsilon)x(\varepsilon)$$

where $0 < \varepsilon \ll 1$ is a small parameter and

$$A(\varepsilon) = \begin{pmatrix} 1-\varepsilon & 2 \\ 8 & 1+2\varepsilon \end{pmatrix}.$$

Using the expansions

$$\begin{aligned} A(\varepsilon) &= A_0 + \varepsilon A_1 + \cdots && (2) \\ x(\varepsilon) &= x_0 + \varepsilon x_1 + \cdots && (3) \\ \lambda(\varepsilon) &= \lambda_0 + \varepsilon \lambda_1 + \cdots && (4) \end{aligned}$$

and the Fredholm alternative, find λ_1 and x_1 for both of the eigenvalues of A. Note x_1 is not unique.

6. [8pt]Consider the perturbed algebraic problem

$$\begin{aligned} x - 2y &= \epsilon(x^2 - y) && (5) \\ 2x - 4y &= \epsilon(x - y^2) && (6) \end{aligned}$$

Use the Fredholm Alternative to find (all) x_0, y_0 in the following expansions of the roots of the system above:

$$\begin{aligned} x(\varepsilon) &= x_0 + \varepsilon x_1 + \cdots && (7) \\ y(\varepsilon) &= y_0 + \varepsilon y_1 + \cdots && (8) \end{aligned}$$

7. [5pt] Determine the normal equations for and least squares solution of

$$Ax = b$$

where

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \quad , \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$