# Math 560: Assignment 4

Due: Nov 17-22, 2016.

1. [5] Prove
$$g(x,x') = -\frac{1}{2\pi} log|x-x'|$$
is a fundamental solution of the 2-dimensional Laplacian
$$\nabla^2 g = \delta(x-x')$$
for $x, x' \in \Omega = \mathbb{R}^2$. Mimic the example on Pg 4 of my posted notes "Greens Functions" and use Green's second identity in the plane.

2. [20] Find the Green's functions for $Lu = f$ where
$$\begin{aligned} Lu &= (x+1)u' + u \quad , \quad u(\infty) = 0 \quad , x \in [0,\infty) \\ Lu &= u'' + u \quad , \quad u(0) = u(\pi) \ , \ u'(0) = u'(\pi) \\ Lu &= \frac{1}{2}x^2u'' + 2xu' + u \quad , \quad u(1) = 0, u(2) + u'(2) = 0 \\ Lu &= u''' \quad , \quad u(0) = u'(0) = u''(0) = 0 \quad , x \in [0,1] \end{aligned}$$

3. [10] Find all the eigenvalues and eigenfunctions for the Sturm Liouville eigenvalue problems:
$$\begin{aligned} u'' &= \lambda u \qquad u'(0) = u'(\pi) = 0 \\ -\frac{d}{dx}\left(x^2u'(x)\right) &= \lambda x^2 u \qquad u(\pi) = u(2\pi) = 0 \end{aligned}$$

4. [5] Use the eigenfunctions of $Lu = u''$ with domain
$$D(L) = \{u \in C^2[0,\pi] : u(0) = u(\pi) = 0\}$$
to find a series representation of the Green's function $g_\lambda(x,t)$ associate with the self adjoint problem
$$(L - \lambda I)u = f \qquad u \in D(L)$$
Here $\lambda$ is any real number not equal to an an eigenvalue and
$$u(x) = (L-\lambda I)^{-1} f = \int_0^\pi g_\lambda(x,t) f(t)\, dt$$