**Math 561: Homework 7.**
Due: Thursday, March 30, 2017.

1. [25] Find two term expansions[1] for all regular solutions of:

$$x^5 + \sin(\epsilon x) - 1 = 0$$

$$\begin{pmatrix} x^3 - y \\ y - x^2 \end{pmatrix} = \epsilon \begin{pmatrix} (y-1)e^x + y \\ x^3 \end{pmatrix}$$

$$y'' - y = \epsilon y^2 \quad , \quad y(0) = 1 \ , \ y(1) = e$$

$$y'' + \epsilon x^2 y = \lambda(\epsilon) y \quad , \quad y(0) = y(\pi) = 0$$

$$y' - y = \epsilon \int_0^1 y(x)^3 dx \quad , \quad y(0) = 1 \ .$$

2. [5] Using $\ll$ notation, asymptotically order the following functions

$$\frac{\sin \epsilon^3}{\epsilon} \ , \ \epsilon \ , \ \frac{1}{\ln(\epsilon)} \ , \ \epsilon^2 e^{-\frac{1}{\epsilon}} \ , \ \epsilon \ln(\epsilon)$$

3. [10] Find the leading asymptotic behavior of $y(x)$ as $x \to \infty$. To do this substitute the assumed forms in the differential equation and choose the constants so that the equation is asymptotically satisfied to leading order.

$$y^3 \ y'' = -\log(x) \qquad y \sim \phi = Ax^\beta (\log x)^\alpha$$
$$x^2 y'' + xy' - (x^2 + n^2) y = 0 \qquad y \sim \phi = Ax^\beta e^x$$

5) [15] Assuming $f \sim a_1 \epsilon^\alpha + a_2 \epsilon^\beta$ where $\alpha < \beta$ and $a_1 a_2 \neq 0$, find $a_1$ and $a_2$ for the following functions:

a) $f(\epsilon) = (2 + \sin(\epsilon))^{3/2}$

b) $f(\epsilon) = \int_0^\epsilon \sin(x + \epsilon^2) dx$

c) $f(\epsilon) = (1 + x\epsilon)^{1/\epsilon}$

[1] first two nonzero terms