Math 561: Homework 8.
Due: Thursday, April 20, 2017.

1. [10] Find leading-order composite asymptotic approximation for the following single boundary layer problem using Prandtl matching.

$$\varepsilon y'' + 2y' + y^3 = 0 \quad , \quad y(0) = 0 \quad , \quad y(1) = 1/2$$

Sketch the solution.

2. [10] Consider the following turning point problem where $a(x) = \sin x$ vanishes.

$$\varepsilon y'' + \sin(x)\, y' + \tan(x) y = 0 \quad , \quad y\left(-\frac{\pi}{4}\right) = \frac{\alpha}{\sqrt{2}-1} \quad , \quad y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}+1}$$

   a) Show that if $\alpha = 1$ the outer solution satifies both boundary conditions so there is no interior layer.
   b) For $\alpha = 2$ find the leading-order composite asymptotic approximation for the interior layer solution (Prandtl matching). Sketch your solution.

3. [10] Find the leading-order composite asymptotic approximation for the interior layer problem:

$$\varepsilon y'' = -yy' + y^3 \quad , \quad y(0) = \frac{1}{2} \quad , \quad y(1) = -\frac{2}{3}$$

Here the layer position $\bar{x}$ is to be found from matching. The leading inner equation is an exact differential. Lastly, use software to graph $y_0^-(x), Y_0(X)$ and $y_0^+(x)$ on the same figure using $\epsilon = 0.001$.