MATH 348 - Sample Test I
September 11, 2015

There is no job so simple that it cannot be done wrong.

1. Answer each of the following T (true) or F (false).
   (a) T F
       y = e^{-x^2} is a solution to \(2\frac{dy}{dx} + y = 0\).
   (b) T F
       y = e^{-x^2} is a solution to \(2\frac{dy}{dx} + xy = 0\).
   (c) T F
       The equation \(\frac{d^2y}{dx^2} + xy^2 = 1\) is linear.
   (d) T F
       The solution to the problem \(\frac{dy}{dx} + x^3y = \cos(2x)\) subject to \(y(0) = 14\)
       has a unique solution.
   (e) T F
       It is possible for \(y_1(x) = e^x\), \(y_2(x) = e^{2x}\) and \(y_3(x) = e^{3x}\) to be
       solutions to \(Ly(x) = 0\).
   (f) T F
       If \(a_1(x) = 0\) and \(a_0(x) = 0\) then the solution of \(Ly(x) = 0\) is \(c_1 + c_2x\).

Now assume \(Ly(x) = y''(x) + a_1(x)y'(x) + a_0(x)y(x)\).
   (e) T F
       It is possible for \(y_1(x) = e^x\), \(y_2(x) = e^{2x}\) and \(y_3(x) = e^{3x}\) to be
       solutions to \(Ly(x) = 0\).
   (f) T F
       If \(a_1(x) = 0\) and \(a_0(x) = 0\) then the solution of \(Ly(x) = 0\) is \(c_1 + c_2x\).

2. Let \(f_1(x) = x\) and \(f_2(x) = |x|\) on the interval \(I = [-1, 1]\).
   (a) Show that the functions \(f_1(x)\) and \(f_2(x)\) are linearly independent on the interval \(I\).
   (b) Find the Wronskian \(f_1(x)\) and \(f_2(x)\) on \(I\).
   (c) Do the results of (a) and (b) contradict Theorem 7 page 513 of your text?

3. The function \(y_h(x) = c_1e^{2x} + c_2e^{-2x}\) is the homogeneous solution to the ode
   \(y''(x) - 4y(x) = 0\).
   (a) Find a particular solution to
   \(y''(x) - 4y(x) = 6x^2 - 8\).
   \(\text{Hint: Set } y_p(x) = Ax^2 + Bx + C\) and use the ode to find \(A, B\) and \(C\)
   (b) Write down the general solution to \(y''(x) - 4y(x) = 6x^2 - 8\).
   (c) Find the unique solution that satisfies \(y(0) = \frac{1}{2}\) and \(y'(0) = \frac{5}{2}\).

4. The equation
   \(\sin(x)y''(x) - 2\cos(x)y'(x) - \sin(x)y(x) = 0\)
models Synchronized Equi-Reciprocal Osmosis or (SERO). Show that \(y_1(x) = \cos(x)\) is
a solution. Find a second independent solution. Hint:
\[\int \tan^2(x) = \tan(x) - x + c = \tan(x) - x\]
upon setting \(c = 0\).