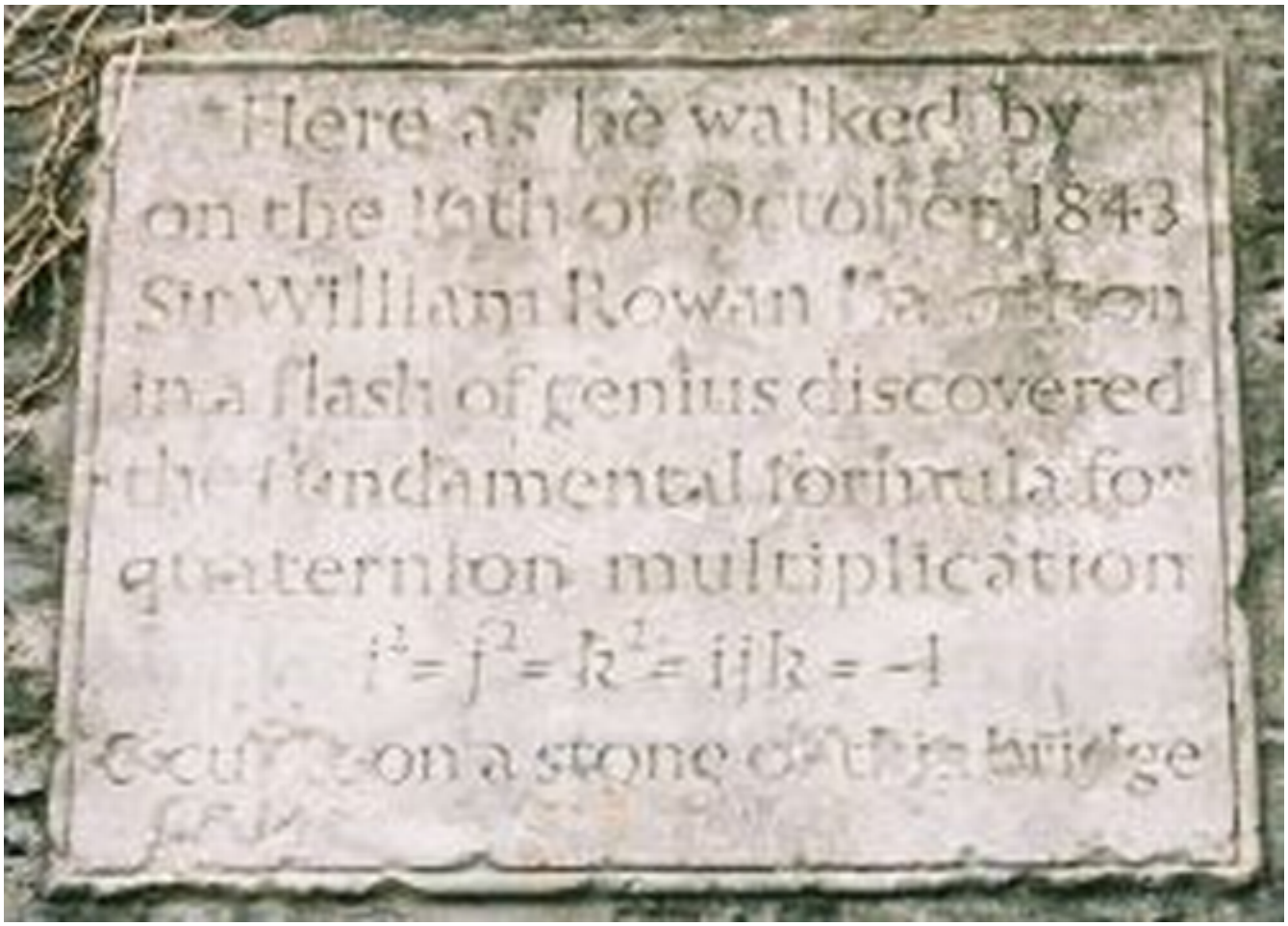
Historical Note on the Scalar and Vector Products

A *complex number* z is an number of the form $z = a + bi$, where $i^2 = -1$ and a and b are real numbers (a being the real part and b the imaginary part of z.) The complex number z may be interpreted as a point in two-dimensional space with coordinates (a, b). In looking for ways to extend the notion of a complex number to higher dimensions, Sir William Rowan Hamilton (1805-1865) developed (or discovered) the concept of a quaternion, $q = a + bi + cj + dk$, on October 16, 1843. In fact on Brougham (Broom) Bridge in Dublin, Ireland, there is a plaque commomorating his discovery of quaternions.



The plaque contains the following inscription: "Here as he walked by on the 16th of October 1843 Sir William Rowan Hamilton in a flash of genius discovered the fundamental formula for quaternion multiplication

$$i^2 = j^2 = k^2 = ijk = -1$$

& cut it on a stone of this bridge."

In a paper published in 1846 he introduces the terms *scalar* and *vector* to refer respectively to the real part a and the imaginary part $bi+cj+dk$ of the quaternion q. If we consider the product of two quaternions each of whose real part is zero, say $q_1 = a_1i+a_2j+a_3k$ and $q_2 = b_1i+b_2j+b_3k$ and apply the laws of quaternion multiplication realizing that $ij = k$, $ji = -k$, $jk = i$, $kj = -i$, $ki = j$, and $ik = -j$, we obtain $q = q_1q_2 = -(a_1b_1 + a_2b_2 + a_3b_3) + (a_2b_3 - a_3b_2)i - (a_1b_3 - a_3b_1)j + (a_1b_2 - a_2b_1)k$. We note that the scalar part of q_1q_2 contains the the negative of the scalar product and the vector part contains the vector product.