Math 221
Computer Exam # 1

Label each problem and each part of each problem by inserting text. Make any comments you need to using the text mode of Maple 11. Since the manual for the class is written using the with(linalg) package this should be the package of choice. As we have been doing in the exercises you may also use the with(LinearAlgebra) package. Make sure you get the response from the packages you expect.

1. The figure Circuits on the web page describes the flow in a circuit. The system of equations is given by looking first at the functions and then at the loop.

   To analyze the current flows I_j, Kirkoff's two laws are used in this regard:
   Kirchoff's current law: At each junction the currect flowing in equals the current flowing out.

   Kirchoff's voltage law The algebraic sum of the voltages drops across resistors plus the algebraic sum of the voltage sources in any closed loop equals zero.
   Ohm's law The voltage drop across a resistor in the direction of the current flow is RI and -RI in the direction opposite to the current flow. The sign attached to a voltage source is positive if the direction of the loop traverses it from - to +; otherwise the voltage source is considered negative.

   Junction equations:
   at B $I_1 = I_2 + I_4$
   at C $I_2 = I_3 + I_5$
   at F $I_4 + I_6 = I_1$
   at E $I_5 + I_3 = I_6$

   Note the first four equation can be combined into three equations:
   1. $I_1 - I_2 - I_4 \ = \ 0$
   2. $I_2 - I_3 - I_5 \ = \ 0$
   3. $-I_1 + I_4 + I_6 \ = \ 0$

   a. Recall that Gauss Elimination is replacing one equation by a linear combination of other equations. A row of zeros at the end of Gauss Elimination tels us that the row with all zeros is a linear combination of the previous row in the origional system. Use this fact to verify the comment that the first four Junction equation can be replaced by the first three equations

   b. Write the equation for the loop equations i.e loop ABFA, BCEFB and CDEC.

   c. Use the equation 1.,2.,3.,ABFA,BCEFB,and CDEC to solve for the currents I_1, I_2 through I_6. List the coefficient equation and the right hand side as 6×6 matrix A and a 6×1 column matrix **b**. Use the augment command to form the matrix $[A|\mathbf{b}]$. Next use gausselim and backsub commands to solve the system of equations. Label the solution as **II**. **Note the letter I is protected in Maple**

   e. Verify your solution by multiplying A by **II** to see if you get **b**.

   f. Use the determinate to determine of the matrix A is non-singular.

   g. Use the rank command on A to determine if the solution is unique.

2. For this problem use the two diagrams found on the web page titles Tempetures 1 and Tempetures 2. They represent the temperatures in a thin metal plate where the temperature on the boundary are described in the diagrams. x

   a. Build a coefficient matrix to solve for the internal temperatures $T_1, ..., T_{16}$. You are to use the block matrix command. Use the symbols S for the diagonal block, I_4 for the 4×4 identity and Z_4 for the 4×4 zero matrix. See the lab manual for the blockmatrix command and how to use it. Place the left hand side values of the system to be solved in the column vector **b**. Since you want a column vector you can use the matrix command to build this 16×1 matrix.

   b. Find the PA=LU decomposition of the matrix A by using the LUdecomposition command page 46 of Maple manual of your text. List and label the matrices P, L and U. Also, blockmatrix command page 25 of the maple manual on how to enter the A matrix into the computer.

   c. Solve the two systems by using first forwardsub and then backsub. **You are not to use Gausselim or GaussJord on the problem. It you do you will receive no credit.**

3. For this problem we will consider the following forces: a force of 20 newtons in the direction from the origin through the point (3,2), a force of 25 newtons in the direction from the origin through the point (-1,3), a force of 10 newtons in the direction from the origin through the point (-4,-3) and a force of 15 newtons in the direction from the origin through the point (2,-1). **Note that the vector [3,2] is not a unit vector. To find a force of 20 in the direction of [3,2] you will need a unit vector in that direction**

   a. Give the algebraic representation of each force as an ordered pair of numbers.

   b. Find the resultant force by finding it magnitude and direction. The direction can be given as a vector through the origin to a point.

   c. Find a force that would make the combination of forces be in equilibrium.

   d. Find the angle between the first two forces above.

4. The city traffic commission has collected data on traffic flow for the street network of one way streets shown below. The direction of the traffic flow between intersections is indicated by the arrows. The intersections are labeled A through F and the average number of vehicles per hour on portions of the streets is indicated by x_1 through x_8. The average number of vehicles that enter or exit a street appears near the street. The vehicles entering an intersection must also exit the intersection, so we can construct an input-output equation for each intersection. The diagram can be found on the web page titled Traffic 1.

   a. Write a system of equation to describe the flow in the diagram. Call the coefficient matrix F and the right hand side c. I only need to see the coefficient matrix F and the right hand side c.

   b. Use the gaussjord command on the augmented matrix of F and c. Determine the rank of the matrix F and the matrix of F augmented with c.

   c. Use backsub to find the solution of the system. Write the solution as a column vector or matrix.

   d. Give three solutions that model a real world flow on the streets. Since these are one way streets the flows will need to all be positive.