Math 221
Test # 2

SHOW ALL WORK FOR CREDIT **Name**____________________________

1. Use gausselim and gaussjord on A and A^T to work parts (a) and (b) of the following problem.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \\ 2 & 4 & 8 \end{bmatrix}$$

and

$$A^T = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 4 \\ 1 & 2 & 8 \end{bmatrix}$$

a. (6 pts) Find two representation for a basis for the row space of A.

b. (6 pts) Find two representation for a basis for the column space of A.

c. (5 pts) Find a basis for the null space of A by adjoining the zero vector to A and using backsub and gauselim to the augmented matrix. .

d. (3 pts) Determine if a set of basis vectors for the row space and the nullspace vectors are linearly independent.

2. Given the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

a. (5 pts) Determine whether the vectors $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ are linearly independent or linearly dependent.

b. (3 pts) Project vector $\mathbf{v}_2$ onto $\mathbf{v}_1$.

c. (5 pts) Use the Gram-Schmidt algorithm not the GramSchmidt command in Maple to build a set of orthogonal vectors using the above vectors starting with $\mathbf{v}_1$

d. (5 pts) Find an orthogonal matrix and an upper triangular matrix such that B=QR where

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

e. (6 pts) Use part c of this problem to solve B$\mathbf{x} = \mathbf{e}$ where $\mathbf{e} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

3. (8 pts) Find the least squares linear fit y=a+bx to the data points (-1,-1),(0,1),and (1,3.1). **Do not use the least squares command from Maple**

4. a. (5 pts) Determine whether the following subset of $\mathcal{R}^4$ is a subspace.

$$\mathcal{S} \;=\; \left\{ \begin{bmatrix} x+y \\ x-y \\ x \\ y \end{bmatrix} |\text{x and y are real numbers} \right\}$$

b. (5 pts) Determine whether the following subset of M_{nn} (the set of all $n \times n$ matrices is a subspace.

$$\mathcal{U} \;=\; \left\{ X|XA = 0\text{where A is a fixed n by n matrix} \right\}$$

5. (8 pts) Find a basis for the following subspace of M_{22} (all 2×2 matrices).

$$\mathcal{N} \;=\; span\left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} , \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\}$$

Also, determine the dimension of the subspace

6. Given the set of vectors

$$\left\{ \mathbf{w}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} , \mathbf{w}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}.$$

a. (7 pts) Use gausselim and backsub to determine the form of the vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ which are in $span\{\mathbf{w}_1, \mathbf{w}_2\}$.

b. (7 pts) Expand the vectors $\mathbf{w}_1, \mathbf{w}_2$ to a basis for $\mathcal{R}^3$.

7. (5 pts). Show that a set of k orthogonal vectors in $\mathcal{R}^n$ where $k < n$ is linearly independent.

8. (10 pts) **True and False** Answer ture if the statement is always true and false otherwise.

   a. T, F A set of r orthogonal vectors in $\mathcal{R}^n$ where $r < n$ is linearly independent.

   b. T, F If $\mathcal{U} = span\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4\}$, then $\dim(\mathcal{U}) = 4$.

   c. T, F If Q is an orthogonal matrix then since $Q^TQ = I$ we know that $Q^T = Q^{-1}$.

   d. T, F A subspace of a vector space is a subset which is closed under the addition and scalar multiplication of the vector space.

   e. T, F A set of m vectors in $\mathcal{R}^n$ where $m > n$ is always linearly dependent.