Selected Solutions for Sections 1.3 and 1.4

Problem 13, page 45.
Show that if the determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0, \tag{1}$$

then the solutions of the equations

$$\begin{aligned} ax + by &= e, \quad (2) \\ cx + dy &= f, \quad (3) \end{aligned}$$

are given by the formulas

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, \qquad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}. \tag{4}$$

This question is asking you to *derive* Cramer's Rule for a 2×2 system. This can be achieved by using the method of elimination. Without loss of generality, we can assume that $a \neq 0$ (if $a = 0$, then the system can be solved directly, and the same result holds). Similarly, if $c = 0$, then $ad \neq 0$ and we can solve the equations directly. Therefore, we concentrate on the case when $ac \neq 0$. Multiply equation (2) by c and equation (3) by $-a$. Adding these two equations together, we eliminate the variable x and are left with a linear equation for y given by

$$(bc - ad)y = ce - af.$$

Then y is given by

$$y = \frac{af - ce}{ad - bc} = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}. \tag{5}$$

Using this result, we can substitute for y into either of equation (2) or (3) to recover a similar expression for x.

Problem 33, page 45.
Find the distance from the point $(2,1,-1)$ to the plane given by $x+2y+2z+5=0$.

The distance can be computed by using the formula on page 57. A normal to the plane is given by $\mathbf{n}=(1,2,2)=\mathbf{i}+2\mathbf{j}+2\mathbf{k}$. Then the distance, D, is given by

$$D=\frac{|1(2)+2(1)+2(-1)+5|}{\sqrt{1^2+2^2+2^2}}=\frac{7}{3}. \tag{6}$$