Name _____ Score _____

Spring 2007

MATH 442 Homework 1

Carefully Read and Follow Directions Clearly label your work and attach it to this sheet. No credit will be given for unsubstantiated answers.

1. Assume that $f: \mathbb{R} \to \mathbb{R}$ is twice continuously differentiable, and let x^* be a point for which
$$f'(x^*) = 0 \quad \text{and} \quad f''(x^*) > 0.$$
Show that there exists an interval I containing x^* for which
$$f(x) < f(x^*) \quad \forall\, x \in I,\ x \neq x^*.$$
(HINT: Consider a quadratic Taylor expansion for f)

2. The **Mean Value Theorem for Integrals** is stated below.

Theorem 1. *Assume that f is continuous on $[a, b]$. Then there exists a point $c \in (a, b)$ so that*
$$\frac{1}{b-a}\int_a^b f(x)dx = f(c).$$
The value $f(c)$ is the average value of f over the interval $[a, b]$.

Use this theorem to find the average area of all circles centered at the origin with radii between 1 and 3. In addition, find the radius of the circle whose area attains the average value you have just computed.

3. Assume that f, f', f'' are continous on the interval $[a, b]$. Assume that $f(a) = f(b) = 0$ and that $f(x) > 0$ for all $x \in (a, b)$. Show that there exists a number $c \in (a, b)$ so that $f''(c) < 0$. (HINT: Rolle's Theorem and Taylor's Theorem)

4. Find the point on the parabola $y = x^2$ that is closest to the point $(1, 0)$. Pose this problem in the form of an optimization problem, and explicitly identify the objective function that you are minimizing.