

### MATH 442 Homework 2

**Carefully Read and Follow Directions** Clearly label your work and attach it to this sheet. No credit will be given for unsubstantiated answers.

1. You are given the following data.

$i$	0	1	2	3
$x_i$	0	2	3	4
$y_i$	7	11	28	63

Let  $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$  denote an interpolating polynomial which passes through the points given in data set above.

- (a) Write a linear system of equations to determine the coefficients  $a_i$  using the relationship that  $p(x_i) = y_i$ . DO NOT SOLVE THE LINEAR SYSTEM.
  - (b) Determine the Lagrange form of the cubic interpolant for the data.
  - (c) Determine the Newton form of the cubic interpolant for the data.
  - (d) For each of the polynomials in parts (b) and (c), write them in the form  $c_0 + c_1x + c_2x^2 + c_3x^3$  to verify that they are identical. Use this information to give the solution to the system in part (a).
2. Given a set of nodes  $x_0 < x_1 < \dots < x_n$  and the corresponding cardinal functions  $\ell_j$  for  $j = 0, 1, \dots, n$ , show that

$$\sum_{j=0}^n \ell_j(x) = 1.$$

Convince yourself for  $n = 1$  and  $n = 2$ . To perform the rigorous argument, consider the polynomial of degree at most  $n$  given by

$$g(x) = \sum_{j=0}^n \ell_j(x) - 1.$$

How many roots does this polynomial  $g(x)$  have (at least)?

3. Suppose  $f(x) = e^{-x}$  is to be approximated by an  $n$ -th degree Lagrange interpolating polynomial  $p_n(x)$  using an evenly spaced partition of nodes on the interval  $[0, 2]$ . How large does  $n$  have to be to guarantee that  $|f(x) - p_n(x)| < 10^{-10}$  on  $[0, 2]$ .