

### MATH 442 Homework 4

**Carefully Read and Follow Directions** Clearly label your work and attach it to this sheet. No credit will be given for unsubstantiated answers.

1. Suppose that you are constructing a quadrature rule to approximate the integral  $\int_{-1}^1 f(x)dx$ . Find the values of  $w_1, w_2, w_3$  and  $w_4$  so that the following quadrature rule has degree of precision 3.

$$Q(f) = w_1f(-1) + w_2f(1) + w_3f'(-1) + w_4f'(1)$$

2. Approximate  $\int_0^2 2^x dx$  (by hand) using 4 subintervals of equal length with each of the following quadrature rules.
  - (a) Composite Trapezoidal Rule
  - (b) Composite Simpson's Rule
  - (c) Give a bound for the amount of truncation error that one can expect for part (b), compute the actual error and compare the two.

3. (a) Using a variable substitution, show that the two integrals are equivalent, that is, show that

$$\frac{1}{\sqrt{2\pi}} \int_0^1 e^{-t^2/2} dt = \frac{1}{\sqrt{2\pi}} \int_{-1}^1 \frac{e^{-(x+1)^2/8}}{2} dx$$

- (b) Approximate the integral in part (a) using a 2 point Gauss quadrature rule.
4. Suppose that you want to use Composite Trapezoidal Rule to approximate the integral

$$\int_0^2 \frac{1}{3x+1}$$

to within  $10^{-5}$ . Determine the value of  $n$  (or  $h$ ) required to do so.