Name ________________________ Score __________________

Spring 2007

MATH 442 Homework 5

Carefully Read and Follow Directions Clearly label your work and attach it to this sheet. No credit will be given for unsubstantiated answers.

1. Given the IVP

$$y' = f(t, y)$$
$$y(t_0) = y_0$$

   Derive the Adams-Bashforth Two-Step Method by the following approach. Find a and b so that the LDE given below is $O(h^3)$.

$$\epsilon_{i+1} = y(t_{i+1}) - [y(t_i) + ahf(t_i, y(t_i)) + bhf(t_{i-1}, y(t_{i-1}))] = O(h^3)$$

   Use Taylor exansions for the terms $y(t_{i+1})$ and $f(t_{i-1}, y(t_{i-1}))$ about the point (t_i, y_i), and force the coefficients of h and h^2 to be zero by choosing a and b appropriately.

2. Show that the solution to initial value problem

$$\begin{aligned} \frac{dy}{dt} &= 100y + e^{-t}, \\ y(0) &= -\frac{1}{101}. \end{aligned}$$

   is unstable with respect to perturbations in the initial data. (Begin by writing the general solution of the ODE, and considering functions $z(t)$ which solve the ODE and satisfy perturbed initial conditions given by $z(0) = -\frac{1}{101} + \delta$.)

3. Download the m-files: *Euler.m*, *EulerDriver.m* and *RK.m*. You will need to create a function file which returns values for the right-hand side of the following ODE. Recall that the initial value problem given by

$$\begin{aligned} \frac{dy}{dt} &= te^{3t} - 2y, \quad 0 \le t \le 1 \\ y(0) &= 0 \end{aligned}$$

   has the exact solution

$$y(t) = \frac{e^{3t}}{5}(t - \frac{1}{5}) + \frac{e^{-2t}}{25},$$

   see class handouts. We begin by using the m-files *Euler.m* and *EulerDriver.m* to numerically approximate the solution to this IVP. Answer the following questions.

   (a) What functions does the driver call? (including the built-in Matlab functions as well as externally defined functions.)

(b) In a brief paragraph, describe the process that the driver *EulerDriver.m* carries out?

(c) What is the initial condition for the initial value problem that EulerDriver is currently solving?

(d) What two pieces of data does the function *Euler.m* return?

After you create the function file (and edit the driver to use the name of the function), run *EulerDriver.m*. Label the plot as **Homework 5 Number 3**, and submit this figure.

4. Suppose that one wants to solve the IVP given by

$$\begin{aligned} \frac{dy}{dt} &= 1 + (t - y)^2, \quad 2 \leq t \leq 3 \\ y(2) &= 1, \end{aligned}$$

which has the true solution $y(t) = t + \frac{1}{1-t}$. Modify *EulerDriver.m* and your function file in order to compute a numerical approximation to this solution using a step size of $h = 0.25$. (With regard to your function file, just comment out the previous function statement, and type a new line for the new ODE.) Remember to save all the files you edit before you run EulerDriver.m for the new IVP. After you run the driver for the $h = 0.25$ case, type **hold on** in the command window. Then modify the driver to use a step size of $h = 0.1$ and run it again. (Also adjust the color of the numerical approximation so that it will plot in a color other than blue.) Label the figure as **Homework 5 Number 4**, and submit it with your work.

5. Copy the code in *EulerDriver.m* into a new file and save it as *RKDriver.m*. Modify this new m-file to use *RK.m* to implement the second order Runge-Kutta method for the IVP in Problem # 4. (See RK.m for more info) You will only need to modify one line of code (other than the commands for your plots). Label the figure as **Homework 5 Number 5**, and compare it with the results you obtained using Euler's method.

6. Copy the code from the function *RK.m* into a new file and save it as *RK4.m*. Edit the new file to implement the 4th order Runge-Kutta method given in your notes. You may use the same driver *RKDriver.m* for the following results and simply modify the function call. Use the 4th order Runge-Kutta method to obtain a numerical approximation to the solution of the IVP given in Problem #4. Generate a figure which shows the approximations for both step sizes $h = 0.25$ and $h = 0.1$, title it **Homework 5 Number 6**. Finally, create a new driver (which will be similar to *RKDriver.m*) in order to generate a table which computes the global error in the numerical approximation of $y(2.5)$ for the step sizes $h = 0.1, 0.01, 0.001, 0.0001$. Use the diary command to record your output in a text file. What kind of reduction of error do you expect to see in the table, and what kind of error do you observe?