Name ________________________ Score __________________

Spring 2007

MATH 442 Final Exam

Carefully Read and Follow Directions Clearly label your work and attach it to this sheet. No credit will be given for unsubstantiated answers.

1. Given the IVP

$$y' = 1 + \frac{y}{t}$$
$$y(1) = 2,$$

   (a) Derive the Taylor Series Method of Order 2 for this IVP.

   (b) Derive the Taylor Series Method of Order 4 for this IVP.

2. Consider the IVP

$$y' = f(t, y)$$
$$y(t_0) = y_0,$$

   (a) Using the derivative approximation,

$$y'(t_{n+1}) \approx \frac{3y(t_{n+1}) - 4y(t_n) + y(t_{n-1})}{2h},$$

   derive the implicit multistep method given by

$$y_{n+1} = \frac{4}{3}y_n - \frac{1}{3}y_{k-1} + \frac{2}{3}hf(t_{n+1}, y_{n+1}).$$

   (b) Compute the Local Discretization Error of the method.

   (c) Is the method Consistent?

   (d) Give the stability polynomial, and show that the method is **strongly stable**.(as defined in class)

   (e) Comment on the convergence properties of this method.

3. (a) Verify that the following quadrature formula for approximating the integral $\int_0^1 f(t)dt$ is exact for polynomials of degree ≤ 4.

$$Q(f) = \frac{1}{90}\left[7f(0) + 32f(\frac{1}{4}) + 12f(\frac{1}{2}) + 32f(\frac{3}{4}) + 7f(1)\right]$$

   (b) Using the formula in the preceding problem, obtain a formula for $\int_a^b f(x)dx$ that is exact for all polynomials of degree ≤ 4.

   (c) Approximate $\ln 2$ by applying the formula to

$$\int_0^1 \frac{dt}{t+1}.$$

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4. Download the following files:

   - Adams-Bashforth Two-Step Method and driver, **AdBash2.m** and **AdBash2Driver.m**
   - Modified Euler method, **ModEul.m**,

   You will also need to use your **RK4.m** function that you built for Homework Assignment #5. **AdBash2Driver.m** is a driver that implements a two-step Adams-Bashforth method with a Modified Euler start up. The Adams-Bashforth iteration is performed within the function **AdBash2.m**, and you will need to create a function file which returns the function values for the particular IVP that you are solving.

   (a) Modify the driver **AdBash2Driver.m** and create a function file to solve the following initial value problem using a step size of $h = 0.2$,

   $$\begin{aligned} \frac{dy}{dt} &= -y \ln y, \quad 0 \leq t \leq 2 \\ y(0) &= 0.5, \end{aligned}$$

   which has true solution $y(t) = e^{(-\ln 2)e^{-t}}$. Don't forget to modify the driver so that the plot reflects the accurate actual solution. Label the figure as **Final Exam, Number 4a**, and submit the figure along with copies of your m-files.

   (b) Copy the driver **AdBash2Driver.m** into a file called **AdBash4Driver.m**, and copy the function **AdBash2.m** into a file called **AdBash4.m**. Modify the files **AdBash4Driver.m** and **AdBash4.m** to implement a four-step Adams-Bashforth method with a fourth-order Runge-Kutta start up. You will use the fourth-order Runge-Kutta function to replace the Modified Euler function to obtain your startup values in the 4-step Adams-Bashforth iteration. Once you have made the modifications, test your new code (and get rid of some of the bugs) by solving the problem in part (a). Save the figure as **Final Exam, Number 4b**. Submit the figure along with copies of your m-files.