Name _____ Score _____

Fall 2008

Math 560 Final

Due Wednesday, December 17, 2008

1. Consider the equation $A\vec{x} = \vec{b}$ given by

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 4 & 5 \\ 3 & 6 & 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

   (a) Find conditions on $\vec{b}$ for which the above system has a solution.

   (b) Find a basis for each of the following subspaces: $\mathcal{R}(A), \mathcal{R}(A^T), \mathcal{N}(A), \mathcal{N}(A^T)$.

   (c) Verify that $\mathcal{R}(A^T)^\perp = \mathcal{N}(A)$ and that $\mathbb{R}^4 = \mathcal{N}(A) \oplus \mathcal{R}(A^T)$.

   (d) Find the general solution of $A\vec{x} = \vec{b}$ if $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

   (e) If $\vec{b} \notin \mathcal{R}(A)$, describe how to find the least-squares solution of $A\vec{x} = \vec{b}$.

2. On page 95 of Keener textbook: Problem Section 2.2, Number 2a.

3. On page 128 of Keener textbook: Problem Section 3.1, Number 1.
   Leibniz Rule: (statement taken from J.D. Logan textbook)

$$\frac{d}{dx}\int_{a(x)}^{b(x)} F(x,y)dy = \int_{a(x)}^{b(x)} F_x(x,y)dy + F(x,b(x))b'(x) - F(x,a(x))a'(x)$$

4. On page 128 of Keener textbook: Problem Section 3.2, Number 2.